

الأستاذ:  
نجيب عثمانى

تمارين: المجموعات والعمليات في مجموعة  
الأعداد الحقيقية  
المستوى : الجذع مشترك علمي و الجذع مشترك تكنولوجيا

أكاديمية  
الجهة الشرقية

$$M = [a - (b - c - 1)] - [(-a - b) - (a - 1 + c)] - (b - c)$$

$$M = (a - b + c + 1) - (-a - b - a + 1 - c) - b + c$$

$$M = a - b + c + 1 + a + b + a - 1 + c - b + c = 3a - b + 3c$$

**تمرين 3:** أحسب وبسط:  $A = \left(-1 + \frac{2}{5} + \frac{1}{4}\right)(-4) + \left(-\frac{3}{4} + \frac{5}{3} - \frac{2}{5}\right)\left(\frac{18}{5}\right)$

$$B = \left(\frac{4}{9} - \frac{11}{27}\right)\left(2 - \frac{4}{3}\right) - \left(\frac{3}{5} - \frac{7}{15}\right)\left(\frac{4}{3} - \frac{1}{2}\right)$$

$$C = \frac{7}{3}\left(\frac{3}{5} - \frac{2}{3} + \frac{3}{4}\right) + \left(-\frac{5}{6} + \frac{2}{3}\right)\left(\frac{1}{6} - \frac{2}{3}\right)$$

$$E = \frac{9 - \frac{1}{3} + \frac{5}{6}}{-5 + \frac{1}{2} - \frac{3}{4}} \times \frac{8 - \frac{1}{5} - \frac{7}{10}}{1 - \frac{3}{2} - \frac{5}{4}} \quad \text{و} \quad D = \frac{\frac{1}{2} - \frac{1}{3} + \frac{2}{5} + \frac{1}{6} + 1}{\frac{1}{2} + \frac{1}{3} - \frac{2}{5} - \frac{5}{6}}$$

$$G = \frac{1 - \frac{1}{3} + \frac{1}{1 + \frac{1}{3}}}{1 + \frac{1}{3} - \frac{1}{1 - \frac{1}{3}}} \quad \text{و} \quad F = 5 + \frac{1}{4 + \frac{1}{3 + \frac{1}{2}}}$$

**الجواب:**  $A = \left(-1 + \frac{2}{5} + \frac{1}{4}\right)(-4) + \left(-\frac{3}{4} + \frac{5}{3} - \frac{2}{5}\right)\left(\frac{18}{5}\right)$

$$A = \left(\frac{-20 + 8 + 4}{20}\right)(-4) + \left(\frac{-27 + 60 - 8}{36}\right)\left(\frac{18}{5}\right)$$

$$A = \left(\frac{-7}{20}\right) \times (-4) + \frac{25}{36} \times \frac{18}{5} = \frac{7}{5} + \frac{5}{2} = \frac{14 + 25}{10} = \frac{39}{10}$$

$$B = \left(\frac{4}{9} - \frac{11}{27}\right)\left(2 - \frac{4}{3}\right) - \left(\frac{3}{5} - \frac{7}{15}\right)\left(\frac{4}{3} - \frac{1}{2}\right)$$

$$B = \frac{12 - 11}{27} \times \frac{6 - 4}{3} - \frac{9 - 7}{15} \times \frac{8 - 3}{6} = \frac{1}{27} \times \frac{2}{3} - \frac{2}{15} \times \frac{5}{6}$$

$$B = \frac{2}{81} - \frac{1}{9} = \frac{2 - 9}{81} = \frac{-7}{81}$$

$$C = \frac{7}{3}\left(\frac{3}{5} - \frac{2}{3} + \frac{3}{4}\right) + \left(-\frac{5}{6} + \frac{2}{3}\right)\left(\frac{1}{6} - \frac{2}{3}\right)$$

$$C = \frac{7}{3} \times \frac{36 - 40 + 45}{60} + \frac{-5 + 4}{6} \times \frac{1 - 4}{6} = \frac{7}{3} \times \frac{41}{60} + \frac{1}{6} \times \frac{3}{6}$$

$$C = \frac{287}{180} + \frac{1}{12} = \frac{287 + 15}{180} = \frac{302}{180} = \frac{151}{90}$$

$$D = \frac{\frac{1}{2} - \frac{1}{3} + \frac{2}{5} + \frac{1}{6} + 1}{\frac{1}{2} + \frac{1}{3} - \frac{2}{5} - \frac{5}{6}} + 1 = \frac{\frac{15 - 10 + 12 + 5}{30} + 1}{\frac{15 + 10 - 12 - 5}{30}} + 1 = \frac{\frac{22}{30} + 1}{\frac{-12}{30}} + 1 = \frac{-22}{12} + \frac{12}{12}$$

$$D = \frac{-22 + 12}{12} = \frac{-10}{12} = \frac{-5}{6}$$

$$E = \frac{9 - \frac{1}{3} + \frac{5}{6}}{-5 + \frac{1}{2} - \frac{3}{4}} \times \frac{8 - \frac{1}{5} - \frac{7}{10}}{1 - \frac{3}{2} - \frac{5}{4}} = \frac{54 - 2 + 5}{6} \times \frac{80 - 2 - 7}{4 - 6 - 5} = \frac{57}{6} \times \frac{71}{-7}$$

$$E = \frac{57}{6} \times \frac{-4}{21} \times \frac{71}{10} \times \frac{-4}{7} = \frac{19}{3} \times \frac{2}{7} \times \frac{71}{5} \times \frac{2}{7} = \frac{5395}{735}$$

**تمرين 1:** باستعمال الرموز:  $\in$ ;  $\notin$ ;  $\subset$ ;  $\supset$ ;  $\emptyset$  املأ الفراغات التالية:  $6 \dots \mathbb{Z}$  و

$$-\frac{2}{3} \dots \mathbb{R}^+ \quad \text{و} \quad \mathbb{N} \dots \mathbb{Q} \quad \text{و} \quad \mathbb{Q} \dots \mathbb{R} \quad \text{و} \quad \sqrt{2} \dots \mathbb{R} \quad \text{و} \quad \sqrt{2} \dots \mathbb{Q} \quad \text{و} \quad \frac{2}{3} \dots \mathbb{Q}$$

$$\text{و} \quad \pi \dots \mathbb{Z} \quad \text{و} \quad \mathbb{Z} \dots \mathbb{Q} \quad \text{و} \quad \mathbb{Q} \dots \mathbb{Z} \quad \text{و} \quad \frac{\sqrt{100}}{5} \dots \mathbb{N} \quad \text{و} \quad \frac{6}{2} \dots \mathbb{N} \quad \text{و} \quad \frac{2}{3} \dots \mathbb{N}$$

$$\text{و} \quad 0 \dots \mathbb{Q}^* \quad \text{و} \quad \frac{7}{3} \dots \mathbb{Q}^{**} \quad \text{و} \quad \sqrt{16} \dots \mathbb{N} \quad \text{و} \quad 0 \dots \mathbb{R}^*$$

$$\mathbb{R}^* \dots \mathbb{R} \quad \text{و} \quad \mathbb{R}^+ \dots \mathbb{R} \quad \text{و} \quad \mathbb{Q} \dots \mathbb{R}^+ \quad \text{و} \quad \mathbb{Q}^* \dots \mathbb{R} \quad \text{و} \quad \mathbb{Q} \dots \mathbb{R}^* \quad \text{و} \quad \{1; 3; -8\} \dots \mathbb{N}$$

**الجواب:**  $6 \in \mathbb{Z}$  و  $\frac{2}{3} \in \mathbb{Q}$  و  $\sqrt{2} \notin \mathbb{Q}$  و  $\sqrt{2} \in \mathbb{R}$  و  $\mathbb{Q} \subset \mathbb{R}$

$$\text{و} \quad \mathbb{N} \subset \mathbb{Q} \quad \text{و} \quad \frac{2}{3} \notin \mathbb{N} \quad \text{و} \quad \frac{6}{2} \in \mathbb{N} \quad \text{و} \quad \frac{\sqrt{100}}{5} \in \mathbb{N}$$

$$\text{و} \quad \pi \notin \mathbb{Z} \quad \text{و} \quad \mathbb{Z} \subset \mathbb{Q} \quad \text{و} \quad \mathbb{Q} \not\subset \mathbb{Z} \quad \text{و} \quad 0 \notin \mathbb{Q}^* \quad \text{و} \quad \frac{7}{3} \notin \mathbb{Q}^{**}$$

$$\sqrt{16} \in \mathbb{N} \quad \text{و} \quad 0 \notin \mathbb{R}^* \quad \text{و} \quad \mathbb{R}^* \subset \mathbb{R} \quad \text{و} \quad \mathbb{R}^+ \subset \mathbb{R} \quad \text{و} \quad \mathbb{Q} \subset \mathbb{R}^+$$

$$\text{و} \quad \mathbb{Q}^* \subset \mathbb{R} \quad \text{و} \quad \{1; 3; -8\} \not\subset \mathbb{N}$$

**تمرين 2:** أحسب وبسط:  $A = \frac{3}{4} + \frac{5}{3} - \frac{7}{6}$  و  $B = \frac{-2}{3} + \frac{7}{6} - \frac{1}{4} - 2$

$$C = \left(\frac{2}{3} - \frac{5}{2}\right)^2 \quad \text{و} \quad D = \frac{5 + \frac{1}{3}}{2 - \frac{3}{2}} \quad \text{و} \quad E = \left(1 - \frac{1}{3}\right)\left(\frac{2}{5} + 1 - \frac{1}{2}\right)$$

$$F = \frac{7 - \frac{4}{\pi}}{12 - 21\pi} \quad \text{و} \quad G = [(a - c) - (a - b)] - [(c - a) + (b - c)]$$

$$M = [a - (b - c - 1)] - [(-a - b) - (a - 1 + c)] - (b - c)$$

**الجواب:**  $A = \frac{3}{4} + \frac{5}{3} - \frac{7}{6} = \frac{9}{12} + \frac{20}{12} - \frac{14}{12} = \frac{9 + 20 - 14}{12} = \frac{15}{12} = \frac{5}{4}$

$$B = \frac{-2}{3} + \frac{7}{6} - \frac{1}{4} - 2 = \frac{-8}{12} + \frac{14}{12} - \frac{3}{12} - \frac{24}{12} = \frac{-8 + 14 - 3 - 24}{12} = \frac{-21}{12} = \frac{-7}{4}$$

$$C = \left(\frac{2}{3} - \frac{5}{2}\right)^2 = \left(\frac{4 - 15}{6}\right)^2 = \left(\frac{-11}{6}\right)^2 = \frac{(-11)^2}{6^2} = \frac{121}{36}$$

$$D = \frac{5 + \frac{1}{3}}{2 - \frac{3}{2}} = \frac{\frac{16}{3}}{\frac{1}{2}} = \frac{16}{3} \times \frac{2}{1} = \frac{32}{3}$$

$$E = \left(1 - \frac{1}{3}\right)\left(\frac{2}{5} + 1 - \frac{1}{2}\right) = \left(\frac{2}{3}\right)\left(\frac{4}{10} + \frac{10}{10} - \frac{5}{10}\right) = \frac{2}{3} \times \frac{9}{10} = \frac{2 \times 3 \times 3}{3 \times 5 \times 2} = \frac{3}{5}$$

$$F = \frac{7 - \frac{4}{\pi}}{12 - 21\pi} = \frac{7\pi - 4}{12 - 21\pi} = \frac{7\pi - 4}{\pi} \times \frac{1}{12 - 21\pi} = \frac{7\pi - 4}{\pi} \times \frac{1}{12 - 21\pi}$$

$$F = \frac{7\pi - 4}{\pi} \times \frac{1}{-3(7\pi - 4)} = -\frac{1}{3\pi}$$

$$G = [(a - c) - (a - b)] - [(c - a) + (b - c)] = (a - c - a + b) - (c - a + b - c)$$

$$G = a - c - a + b - c + a - b + c = a - c$$

$$E = \frac{3+2\sqrt{15}+5-(3-2\sqrt{15}+5)}{(\sqrt{3})^2-(\sqrt{5})^2} = \frac{3+2\sqrt{15}+5-3+2\sqrt{15}-5}{(\sqrt{3})^2-(\sqrt{5})^2} = \frac{4\sqrt{15}}{-2} = -2\sqrt{15}$$

$$F = (-\sqrt{10}) \times (-\sqrt{10}) + \sqrt{5} \times (-\sqrt{20}) - \sqrt{2} \times \sqrt{32}$$

$$F = (-\sqrt{10})^2 - \sqrt{5} \times \sqrt{5 \times 4} - \sqrt{2} \times \sqrt{2 \times 16} = 10 - \sqrt{5}^2 \times 2 - \sqrt{2}^2 \times 4$$

$$F = 10 - 10 - 8 = -8$$

$$G = \sqrt{3} \times 2\sqrt{3} - 8\sqrt{8} \times 2\sqrt{2} - 3\sqrt{5} \times \sqrt{20} + \sqrt{2} \times \sqrt{3} \times \sqrt{24}$$

$$G = 2 \times \sqrt{3}^2 - 8\sqrt{8} \times \sqrt{8} - 3\sqrt{5} \times \sqrt{5 \times 4} + \sqrt{2} \times \sqrt{3} \times \sqrt{2 \times 2 \times 3}$$

$$G = 2 \times 3 - 8\sqrt{8}^2 - 3\sqrt{5}^2 \times 2 + \sqrt{2}^2 \times \sqrt{3}^2 \times 2$$

$$G = 6 - 64 - 30 + 12 = -76$$

$$H = 5\sqrt{12} + 8\sqrt{27} + \sqrt{75} - 2\sqrt{48} - \sqrt{147}$$

$$H = 5\sqrt{4 \times 3} + 8\sqrt{9 \times 3} + \sqrt{25 \times 3} - 2\sqrt{16 \times 3} - \sqrt{49 \times 3}$$

$$H = 10\sqrt{3} + 24\sqrt{3} + 5\sqrt{3} - 8\sqrt{3} - 7\sqrt{3} = 24\sqrt{3}$$

**تمرين 6:** بين أن العدد  $E = \frac{5\sqrt{7}}{\sqrt{2}-\sqrt{7}} + \frac{5\sqrt{2}}{\sqrt{2}+\sqrt{7}}$  عدد صحيح نسبي

**الجواب:**  $E = \frac{5\sqrt{7}}{\sqrt{2}-\sqrt{7}} + \frac{5\sqrt{2}}{\sqrt{2}+\sqrt{7}} = \frac{(5\sqrt{7})(\sqrt{2}+\sqrt{7}) + 5\sqrt{2}(\sqrt{2}-\sqrt{7})}{(\sqrt{2}+\sqrt{7})(\sqrt{2}-\sqrt{7})}$

$$E = \frac{5\sqrt{7}\sqrt{2} + 5\sqrt{7}\sqrt{7} + 5\sqrt{2}\sqrt{2} - 5\sqrt{2}\sqrt{7}}{(\sqrt{2})^2 - (\sqrt{7})^2} = \frac{35 + 10}{(\sqrt{2})^2 - (\sqrt{7})^2} = \frac{45}{-5} = -9 \in \mathbb{Z}$$

**تمرين 7:** اجعل مقامات الأعداد التالية جذرية :

$$C = \frac{\sqrt{5} + 3\sqrt{7}}{2\sqrt{5} - 4\sqrt{7}} \quad \text{و} \quad B = \frac{2 - \sqrt{3}}{\sqrt{2} - 3} \quad \text{و} \quad A = \frac{2 + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$$

**الجواب:**  $A = \frac{2 + \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{(2 + \sqrt{3})(\sqrt{2} + \sqrt{3})}{(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})} = \frac{(2 + \sqrt{3})(\sqrt{2} + \sqrt{3})}{(\sqrt{2})^2 - (\sqrt{3})^2}$

$$A = \frac{2\sqrt{2} + 2\sqrt{3} + \sqrt{6} + (\sqrt{3})^2}{2 - 3} = \frac{2\sqrt{2} + 2\sqrt{3} + \sqrt{6} + 3}{-1}$$

$$A = -(2\sqrt{2} + 2\sqrt{3} + \sqrt{6} + 3)$$

$$B = \frac{2 - \sqrt{3}}{\sqrt{2} - 3} = \frac{(2 - \sqrt{3})(\sqrt{2} + 3)}{(\sqrt{2} - 3)(\sqrt{2} + 3)} = \frac{2\sqrt{2} + 6 - \sqrt{6} - 3\sqrt{3}}{(\sqrt{2})^2 - (3)^2}$$

$$B = \frac{2\sqrt{2} - \sqrt{6} - 3\sqrt{3} + 6}{2 - 9} = \frac{\sqrt{6} - 2\sqrt{2} + 3\sqrt{3} - 6}{7}$$

$$C = \frac{\sqrt{5} + 3\sqrt{7}}{2\sqrt{5} - 4\sqrt{7}} = \frac{(\sqrt{5} + 3\sqrt{7})(2\sqrt{5} + 4\sqrt{7})}{(2\sqrt{5} - 4\sqrt{7})(2\sqrt{5} + 4\sqrt{7})}$$

$$C = \frac{\sqrt{5} + 3\sqrt{7}}{2\sqrt{5} - 4\sqrt{7}} = \frac{10 + 4\sqrt{35} + 6\sqrt{35} + 84}{(2\sqrt{5})^2 - (4\sqrt{7})^2} = \frac{94 + 10\sqrt{35}}{20 - 112}$$

$$C = -\frac{47 + 5\sqrt{35}}{46}$$

**تمرين 8:** نعتبر الأعداد التالية :

$$c = \frac{4}{\sqrt{3} - \sqrt{11}} \quad \text{و} \quad b = \frac{2}{\sqrt{7} - \sqrt{3}} \quad \text{و} \quad a = \frac{2}{\sqrt{11} - \sqrt{7}}$$

(1) اجعل مقامات الأعداد جذرية

(2) بسط العدد :  $A = \frac{2}{\sqrt{11} - \sqrt{7}} + \frac{2}{\sqrt{7} - \sqrt{3}} + \frac{4}{\sqrt{3} - \sqrt{11}}$

**الجواب:**  $a = \frac{2}{\sqrt{11} - \sqrt{7}} = \frac{2(\sqrt{11} + \sqrt{7})}{(\sqrt{11} - \sqrt{7})(\sqrt{11} + \sqrt{7})} = \frac{2(\sqrt{11} + \sqrt{7})}{(\sqrt{11})^2 - (\sqrt{7})^2}$

$$a = \frac{2(\sqrt{11} + \sqrt{7})}{11 - 7} = \frac{2(\sqrt{11} + \sqrt{7})}{4} = \frac{\sqrt{11} + \sqrt{7}}{2}$$

$$F = 5 + \frac{1}{4 + \frac{1}{3 + \frac{1}{2}}} = 5 + \frac{1}{4 + \frac{2}{7}} = 5 + \frac{7}{30} = \frac{150 + 7}{30} = \frac{157}{30}$$

$$G = \frac{1 - \frac{1}{3} + \frac{1}{1 + \frac{1}{3}}}{1 + \frac{1}{3} - \frac{1}{1 - \frac{1}{3}}} = \frac{\frac{3-1}{3} + \frac{3}{3+1}}{\frac{3+1}{3} - \frac{3}{3-1}} = \frac{\frac{2}{3} + \frac{3}{4}}{\frac{4}{3} - \frac{3}{2}} = \frac{\frac{17}{12}}{-\frac{1}{6}} = \frac{17}{12} \times -6 = -\frac{17}{2}$$

**تمرين 4:** ليكن  $a$  و  $b$  عددين حقيقيين بحيث :  $a - b = -\frac{7}{6}$

أحسب ما يلي :  $A_1 = a - \left(b - \frac{71}{61}\right)$  و  $A_2 = \left(a - \frac{9}{5}\right) - \left(b - \frac{9}{5}\right)$

$A_3 = \left(b + \frac{2016}{2017}\right) - \left(a - \frac{1}{2017}\right)$  و  $A_4 = (2a - 5) + (6 - 2b)$

**الجواب:**  $A_1 = a - \left(b - \frac{71}{61}\right) = a - b + \frac{71}{61} = -\frac{7}{6} + \frac{71}{61} = -\frac{853}{336}$

$$A_2 = \left(a - \frac{9}{5}\right) - \left(b - \frac{9}{5}\right) = a - \frac{9}{5} - b + \frac{9}{5} = a - b = -\frac{7}{6}$$

$$A_3 = \left(b + \frac{2016}{2017}\right) - \left(a - \frac{1}{2017}\right) = b - a + \frac{2016}{2017} + \frac{1}{2017}$$

$$A_3 = -(a - b) + \frac{2016 + 1}{2017} = -\left(-\frac{7}{6}\right) + \frac{2017}{2017} = \frac{7}{6} + 1 = \frac{7}{6} + \frac{6}{6} = \frac{13}{6}$$

$$A_4 = (2a - 5) + (6 - 2b) = 2a - 5 + 6 - 2b = 2a - 2b + 1$$

$$A_4 = 2(a - b) + 1 = 2 \times -\frac{7}{6} + 1 = -\frac{7}{3} + 1 = -\frac{4}{3}$$

**تمرين 5:** بسط  $A = \sqrt{\frac{9}{2}}$  و  $B = \frac{\sqrt{28}}{\sqrt{14}}$

$$C = 3\sqrt{20} + 4\sqrt{45} - 2\sqrt{80} - \sqrt{180}$$

$$E = \frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} - \sqrt{5}} - \frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} + \sqrt{5}} \quad \text{و} \quad D = (\sqrt{3} + \sqrt{2} - \sqrt{5})(\sqrt{3} + \sqrt{2} + \sqrt{5})$$

$$F = (-\sqrt{10}) \times (-\sqrt{10}) + \sqrt{5} \times (-\sqrt{20}) - \sqrt{2} \times \sqrt{32}$$

$$G = \sqrt{3} \times 2\sqrt{3} - 8\sqrt{8} \times 2\sqrt{2} - 3\sqrt{5} \times \sqrt{20} + \sqrt{2} \times \sqrt{3} \times \sqrt{24}$$

$$H = 5\sqrt{12} + 8\sqrt{27} + \sqrt{75} - 2\sqrt{48} - \sqrt{147}$$

**الجواب:**  $A = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{(\sqrt{2})^2} = \frac{3\sqrt{2}}{2}$

$$B = \frac{\sqrt{28}}{\sqrt{14}} = \sqrt{\frac{28}{14}} = \sqrt{2}$$

$$C = 3\sqrt{20} + 4\sqrt{45} - 2\sqrt{80} - \sqrt{180} = 3\sqrt{4 \times 5} + 4\sqrt{9 \times 5} - 2\sqrt{16 \times 5} - \sqrt{36 \times 5}$$

$$C = 3 \times 2\sqrt{5} + 4 \times 3\sqrt{5} - 2 \times 4\sqrt{5} - 6\sqrt{5} = 6\sqrt{5} + 12\sqrt{5} - 8\sqrt{5} - 6\sqrt{5} = (6 + 12 - 8 - 6)\sqrt{5}$$

$$C = 4\sqrt{5}$$

$$D = (\sqrt{3} + \sqrt{2} - \sqrt{5})(\sqrt{3} + \sqrt{2} + \sqrt{5}) = ((\sqrt{3} + \sqrt{2}) - \sqrt{5})((\sqrt{3} + \sqrt{2}) + \sqrt{5})$$

$$D = (\sqrt{3} + \sqrt{2})^2 - (\sqrt{5})^2 = (\sqrt{3})^2 + 2\sqrt{3}\sqrt{2} + (\sqrt{2})^2 - 5 = 3 + 2\sqrt{3 \times 2} + 2 - 5$$

$$D = 2\sqrt{6}$$

$$E = \frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} - \sqrt{5}} - \frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} + \sqrt{5}} = \frac{(\sqrt{3} + \sqrt{5})(\sqrt{3} + \sqrt{5}) - (\sqrt{3} - \sqrt{5})(\sqrt{3} - \sqrt{5})}{(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})}$$

$$E = \frac{(\sqrt{3} + \sqrt{5})^2 - (\sqrt{3} - \sqrt{5})^2}{(\sqrt{3})^2 - (\sqrt{5})^2} = \frac{(\sqrt{3})^2 + 2\sqrt{3}\sqrt{5} + (\sqrt{5})^2 - ((\sqrt{3})^2 - 2\sqrt{3}\sqrt{5} + (\sqrt{5})^2)}{(\sqrt{3})^2 - (\sqrt{5})^2}$$

•  $3,25 \times 10^4$  هي كتابة علمية صحيحة  
 •  $15 \times 10^3$  هي كتابة علمية خطأ  
 •  $-1.7 \times 10^7$  هي كتابة علمية صحيحة  
**تمرين 13:** حدد الكتابة العلمية للأعداد التالية  
 0,001 و 0,01 و 45 000 0 و 300 000  
 و 0,0002 و 25 000 000 و 368 100 000 0  
**الجواب:**  $450000 = 4.5 \times 10^5$  و  $300000 = 3 \times 10^5$  و  $0.01 = 10^{-2}$   
 و  $3681000000 = 3.681 \times 10^9$  و  $0.001 = 10^{-3}$   
 و  $0.0002 = 2 \times 10^{-4}$  و  $25000000 = 2.5 \times 10^7$

**تمرين 14:** بسط أو أكتب على شكل قوى :

$$B = (-3)^1 \times (-3)^5 \times (3)^2 \times (-3)^{-10} \quad A = 2^3 \times (2^2)^4 \times (2^{-5})^3$$

$$D = \frac{(-2)^3 \times (4^2)^{-1} \times 8}{1024 \times (-16)^{-4}} \quad C = \frac{3^{-5} \times 4^{-2} \times 9}{12^3} \times \frac{9}{2^2}$$

$$F = \frac{3 \times 10^{-5} \times 7,2 \times 10^7}{2 \times 15^3} \quad \text{و} \quad E = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5}$$

**الجواب:**  $A = 2^3 \times (2^2)^4 \times (2^{-5})^3 = 2^3 \times 2^{2 \times 4} \times 2^{-5 \times 3} = 2^{3+8-15} = 2^{-4}$

$$A = \frac{1}{2^4} = \frac{1}{16}$$

$$B = (-3)^1 \times (-3)^5 \times (3)^2 \times (-3)^{-10} = -(-3)^1 \times (-3)^5 \times (3)^2 \times (3)^{-10}$$

$$B = 3^1 \times 3^5 \times 3^2 \times 3^{-10} = 3^{1+5+2-10} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$C = \frac{3^{-5} \times 4^{-2} \times 9}{12^3} \times \frac{9}{2^2} = \frac{3^{-5} \times (2^2)^{-2} \times 3^2}{(3 \times 2^2)^3} \times \frac{3^2}{2^2} = \frac{3^{-5} \times (2)^{-4} \times 3^2}{(3)^3 \times 2^6 \times 2^2}$$

$$C = \frac{3^{-5} \times (2)^{-4} \times 3^2}{(3)^3 \times 2^6 \times 2^2} = 3^{-5+2} \times 2^{-4-6-2} = 3^{-3} \times 2^{-12} = \frac{1}{3^3 \times 2^{12}}$$

$$C = 3^{-6} \times 2^{-12}$$

$$D = \frac{(-2)^3 \times (4^2)^{-1} \times 8}{1024 \times (-16)^{-4}} = \frac{-2^3 \times 4^{2 \times (-1)} \times 2^3}{1024 \times (-2^3)^{-4}} = \frac{-2^3 \times (2^2)^{-2} \times 2^3}{2^{10} \times (-2^3)^{-4}}$$

$$D = -2^3 \times (2^2)^{-2} \times 2^3 \times 2^{-10} \times (-2)^{3 \times 4} = -2^{3-4+3-10+12} = -2^4 = -16$$

$$E = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5} = 10^{-8+9+7-4-2-3-5} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

$$E = 10^{-8+9+7-4+2-3-5} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

$$F = \frac{3 \times 10^{-5} \times 7,2 \times 10^7}{2 \times 15^3} = \frac{3 \times 10^{-5} \times 3^2 \times 2^3 \times 10^{-1} \times 10^7}{2 \times (3 \times 5)^3}$$

$$F = \frac{3 \times 3^2 \times 2^3 \times 10}{2 \times 3^3 \times 5^3} = \frac{3 \times 3^2 \times 2^3 \times 2 \times 5}{2 \times 3^3 \times 5^3} = \frac{2^3}{5^2} = \frac{8}{25}$$

**تمرين 15:** أحسب وبسط:  $a$  عدد حقيقي غير منعدم

$$A = (\sqrt{2})^{-2} \times (\sqrt{2})^2 \times (-\sqrt{2})^{-5} \times (\sqrt{2})^3$$

$$C = \left( \left( -\frac{3}{2} \right)^{-1} \right)^4 \quad \text{و} \quad B = \left( (-\sqrt{3})^{-2} \right)^2$$

$$E = \left( \frac{a \times (a^{-3})^{-2}}{a^{-2} \times (a^4 \times a^7)^2} \right)^{-3} \quad \text{و} \quad D = \frac{a^{-2} \times (-a)^5 \times a^{-1} \times (a^{-2})^5}{-a \times a^{-4} \times ((-a)^4)^{-2}}$$

$$G = \left( \frac{5^3 \times 2^{-3}}{4 \times 25} \right)^2 \times \frac{2^8}{10^2 \times 5} \quad \text{و} \quad F = \left( -\frac{1}{8} \right)^2 \times \left( \frac{2}{5} \right)^6 \times \left( -\frac{5}{2} \right)^3$$

**الجواب:**  $A = (\sqrt{2})^{-2} \times (\sqrt{2})^2 \times (-\sqrt{2})^{-5} \times (\sqrt{2})^3$

$$b = \frac{2}{\sqrt{7}-\sqrt{3}} = \frac{2(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} = \frac{2(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2 - (\sqrt{3})^2}$$

$$b = \frac{2(\sqrt{7}+\sqrt{3})}{7-3} = \frac{2(\sqrt{7}+\sqrt{3})}{4} = \frac{\sqrt{7}+\sqrt{3}}{2}$$

$$c = \frac{4}{\sqrt{3}-\sqrt{11}} = \frac{4(\sqrt{3}+\sqrt{11})}{(\sqrt{3}-\sqrt{11})(\sqrt{3}+\sqrt{11})} = \frac{2(\sqrt{3}+\sqrt{11})}{(\sqrt{3})^2 - (\sqrt{11})^2}$$

$$c = \frac{2(\sqrt{3}+\sqrt{11})}{3-11} = \frac{2(\sqrt{3}+\sqrt{11})}{-8} = -\frac{\sqrt{3}+\sqrt{11}}{4}$$

(2) بسط العدد  $A$  :

$$A = \frac{\sqrt{11}+\sqrt{7}}{2} + \frac{\sqrt{7}+\sqrt{3}}{2} - \frac{\sqrt{7}+\sqrt{11}}{2} \text{ يعني } A = a+b+c$$

$$A = \frac{\sqrt{11}+\sqrt{7}+\sqrt{7}+\sqrt{3}-\sqrt{3}-\sqrt{11}}{2} = \frac{2\sqrt{7}}{2} = \sqrt{7}$$

**تمرين 9:** (1) أكتب العدد :  $\sqrt{2}+1$  على شكل كسر مقامه عدد جذري

(2) استنتج تبسيطا للعدد :  $G = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$  (3) بسط العدد  $H = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$

**الجواب: (1)**

$$\frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{(\sqrt{2}+\sqrt{1})^2}{(\sqrt{2}-\sqrt{1})(\sqrt{2}+\sqrt{1})} = \frac{3+2\sqrt{2}}{(\sqrt{2})^2 - (\sqrt{1})^2} = 3+2\sqrt{2}$$

$$G = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \sqrt{3+2\sqrt{2}} = \sqrt{(\sqrt{2}+1)^2} = |\sqrt{2}+1| = \sqrt{2}+1 \quad (2)$$

$$H = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \frac{1}{G} = \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{\sqrt{2}-1}{(\sqrt{2})^2 - 1^2} = \sqrt{2}-1 \quad (3)$$

**تمرين 10:** أحسب وبسط  $A = \sqrt{2-\sqrt{2+\sqrt{2}}} \times \sqrt{2+\sqrt{2+\sqrt{2}}} \times \sqrt{2+\sqrt{2}} \times \sqrt{2}$

**الجواب:**  $A = \sqrt{(2-\sqrt{2+\sqrt{2}})(2+\sqrt{2+\sqrt{2}})} \times \sqrt{2+\sqrt{2}} \times \sqrt{2}$

$$A = \sqrt{2^2 - (\sqrt{2+\sqrt{2}})^2} \times \sqrt{2+\sqrt{2}} \times \sqrt{2} = \sqrt{4 - (2+\sqrt{2})} \times \sqrt{2+\sqrt{2}} \times \sqrt{2}$$

$$A = \sqrt{2-\sqrt{2}} \times \sqrt{2+\sqrt{2}} \times \sqrt{2} = \sqrt{2^2 - (\sqrt{2})^2} \times \sqrt{2} = \sqrt{2} \times \sqrt{2} = 2$$

**تمرين 11:** أحسب وبسط:  $A = \frac{(10^6)^4 \times 10^{-2}}{10^4 \times 10^6 \times 10^{-13}}$

$$B = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5}$$

**الجواب:**  $A = \frac{(10^6)^4 \times 10^{-2}}{10^4 \times 10^6 \times 10^{-13}} = 10^{24} \times 10^{-2} \times 10^{-4} \times 10^6 \times 10^{13} = 10^{-1} = \frac{1}{10} = 0.1$

$$B = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5} = 10^{-8+9+7-4-2-3-5} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

**تمرين 12:** أجب بصحيح أو خطأ

• الكتابة العلمية للعدد : 149597870 كلم هي  $1,4959787 \times 10^8$  كلم.

•  $3,25 \times 10^4$  هي كتابة علمية

•  $15 \times 10^3$  هي كتابة علمية

• الكتابة العلمية للعدد : -17000000 هي  $-1.7 \times 10^7$

**الجواب:** الكتابة العلمية تكون على الشكل  $x = a \times 10^p$  أو  $x = -a \times 10^p$  حيث  $p$  ينتمي الى  $\mathbb{Z}$  و  $a$  عدد عشري بحيث  $1 \leq a < 10$  ومنه

•  $1,4959787 \times 10^8$  هي كتابة علمية صحيحة

$$D = 27x^3 - 54x^2 + 36x - 8$$

$$E = (x+2)(x^2 - 2x + 4) = (x+2)(x^2 - 2 \times x + 2^2) = x^3 + 2^3 = x^3 + 8$$

$$F = (200520052006)^2 - (200520052005 \times 200520052007): \text{حساب}$$

نلاحظ أن الأعداد الثلاثة تختلف فقط في رقم وحداتها لتبسيط الحساب

$$x = 200520052006: \text{نضع}$$

$$200520052007 = x + 1 \text{ و } 200520052005 = x - 1: \text{إذن}$$

$$F = x^2 - (x-1)(x+1): \text{منه}$$

$$F = x^2 - (x^2 - 1) = x^2 - x^2 + 1 = 1$$

**تمرين 18:** أنشر وأحسب وبسط حيث  $x \in \mathbb{R}$

$$B = (4\sqrt{3} - 7)^{2015} \times (4\sqrt{3} + 7)^{2015} \text{ و } A = (3 + \sqrt{11})^2 - (3 - \sqrt{11})^2$$

$$D = (5x + 2)^3 \quad C = (\sqrt{75} - \sqrt{98}) \times (5\sqrt{3} + 7\sqrt{2})$$

$$F = (2x - 3)(4x^2 + 6x + 9) \quad E = (\sqrt{3} - 1)^3$$

$$G = (2015200052004)^2 - (2015200052002 \times 2015200052006)$$

$$M = (x^2 - 2x + 1)^2 \text{ و } H = \left(\frac{x}{2} + 2\sqrt{3}\right)^2 + \left(x\sqrt{5} - \frac{3}{2}\right)^2$$

$$R = \left(x^3 + \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2} - x^3\right) \text{ و } N = (x\sqrt{2} + \sqrt{5})(\sqrt{5} - x\sqrt{2})$$

$$L = (3x + \sqrt{2} - \sqrt{5})(3x + \sqrt{2} + \sqrt{5})$$

**الجواب:**

$$A = (\sqrt{3} + \sqrt{11})^2 - (\sqrt{3} - \sqrt{11})^2 = (\sqrt{3})^2 + 2\sqrt{3}\sqrt{11} + (\sqrt{11})^2 - ((\sqrt{3})^2 - 2\sqrt{3}\sqrt{11} + (\sqrt{11})^2)$$

$$A = 3 + 2\sqrt{33} + 11 - (3 - 2\sqrt{33} + 11) = 3 + 2\sqrt{33} + 11 - 3 + 2\sqrt{33} - 11 = 4\sqrt{33}$$

$$B = ((4\sqrt{3} - 7)(4\sqrt{3} + 7))^{2015} = ((4\sqrt{3})^2 - (7)^2)^{2015} = (48 - 49)^{2015} = (-1)^{2015} = -1$$

$$C = (\sqrt{75} - \sqrt{98}) \times (5\sqrt{3} + 7\sqrt{2}) = (\sqrt{25 \times 3} - \sqrt{49 \times 2}) \times (5\sqrt{3} + 7\sqrt{2})$$

$$C = (5\sqrt{3} - 7\sqrt{2}) \times (5\sqrt{3} + 7\sqrt{2}) = (5\sqrt{3})^2 - (7\sqrt{2})^2 = 75 - 98 = 75 - 98 = -23$$

$$D = (5x + 2)^3 = (5x)^3 + 3(5x)^2 \times 2 + 3 \times 5x \times (2)^2 + (2)^3$$

$$D = 125x^3 + 150x^2 + 60x + 8$$

$$E = (\sqrt{3} - 1)^3 = (\sqrt{3})^3 - 3(\sqrt{3})^2 \times 1 + 3 \times \sqrt{3} \times (1)^2 - (1)^3$$

$$E = 3\sqrt{3} - 9 + 3\sqrt{3} - 1 = 6\sqrt{3} - 10$$

$$F = (2x - 3)(4x^2 + 6x + 9) = (2x - 3)((2x)^2 + 2x \times 3 + 3^2) = (2x)^3 - 3^3 = 8x^3 - 27$$

$$G = (2015200052004)^2 - (2015200052002 \times 2015200052006): \text{حساب}$$

نلاحظ أن الأعداد الثلاثة تختلف فقط في رقم وحداتها لتبسيط الحساب

$$x = 2015200052004: \text{نضع}$$

$$x + 2 = 2015200052006 \text{ و } x - 2 = 2015200052002: \text{إذن}$$

$$G = x^2 - (x-2)(x+2): \text{منه}$$

$$G = x^2 - (x^2 - 4) = x^2 - x^2 + 4 = 4$$

$$H = \left(\frac{x}{2} + 2\sqrt{3}\right)^2 + \left(x\sqrt{5} - \frac{3}{2}\right)^2 =$$

$$H = \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times 2\sqrt{3} + (2\sqrt{3})^2 + (x\sqrt{5})^2 - 2x\sqrt{5} \times \frac{3}{2} + \left(\frac{3}{2}\right)^2$$

$$H = \frac{x^2}{4} + 2x\sqrt{3} + 12 + 5x^2 - 3x\sqrt{5} + \frac{9}{4}$$

$$H = \frac{21}{4}x^2 + (2\sqrt{3} - 3\sqrt{5})x + \frac{57}{4}$$

$$M = (x^2 - 2x + 1)^2 = ((x^2 - 2x) + 1)^2$$

$$A = -(\sqrt{2})^{-(2+2+5+3)} = -(\sqrt{2})^{-2} = -\frac{1}{(\sqrt{2})^2} = -\frac{1}{2}$$

$$B = \left((- \sqrt{3})^{-2}\right)^2 = \left(\frac{1}{\sqrt{3}^2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$C = \left(\left(-\frac{3}{2}\right)^{-1}\right)^4 = \left(-\frac{3}{2}\right)^{-4} = \left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = 2^4 \times 3^{-4}$$

$$D = \frac{a^{-2} \times (-a)^5}{-a \times a^{-4}} \times \frac{a^{-1} \times (a^{-2})^5}{((-a)^4)^{-2}} = \frac{-a^{-2} \times a^5 \times a^{-1} \times a^{-10}}{-a \times a^{-4} \times a^{-8}}$$

$$D = \frac{a^{(-2+5-1-10)}}{a^{(-1+4-8)}} = \frac{a^{-8}}{a^{-11}} = a^{-8+11} = a^3$$

$$E = \left(\frac{a \times (a^3)^{-2}}{a^{-2} \times (a^{-4} \times a^7)^2}\right)^3 = \left(\frac{a \times a^6}{a^{-2} \times a^6}\right)^3 = \left(\frac{a}{a^{-2}}\right)^3 = (a \times a^2)^3$$

$$E = (a^3)^{-3} = a^{-9}$$

$$F = \left(-\frac{1}{8}\right)^2 \times \left(\frac{2}{5}\right)^6 \times \left(-\frac{5}{2}\right)^3 = -\left(\frac{1}{2^3}\right)^2 \times \frac{2^6}{5^6} \times \frac{5^3}{2^3} = -\frac{2^6 \times 5^3}{2^6 \times 5^6 \times 2^3}$$

$$F = -\frac{2^6 \times 5^3}{2^9 \times 5^6} = -\frac{1}{2^3 \times 5^3} = -\frac{1}{(2 \times 5)^3} = -\frac{1}{10^3} = -10^{-3}$$

$$G = \left(\frac{5^3 \times 2^{-3}}{4 \times 25}\right)^2 \times \frac{2^8}{10^2 \times 5} = \frac{5^6 \times 2^{-6}}{(2^2 \times 5^2)^2} \times \frac{2^8}{(2^2 \times 5^2) \times 5}$$

$$G = \frac{5^6 \times 2^{-6} \times 2^8}{2^4 \times 5^4 \times 2^2 \times 5^2 \times 5} = \frac{5^6 \times 2^2}{2^6 \times 5^7} = \frac{1}{2^4 \times 5} = \frac{1}{80}$$

**تمرين 16:**  $a$  و  $b$  عدنان حقيقيان غير منعدمين

$$C = \frac{(ab^2)^3 \times a^4 b^2}{(ab)^5}: \text{نعتبر العدد الحقيقي}$$

(1) أحسب وبسط  $C$  (2) أكتب العدد  $C$  على شكل قوة أساسها 10 علما أن:

$$b = 100 \text{ و } a = \frac{1}{10}$$

**الجواب: (1)**

$$C = \frac{(ab^2)^3 \times a^4 b^2}{(ab)^5} = \frac{a^3 \times b^6 \times a^4 \times b^2}{a^5 \times b^5} = \frac{a^7 \times b^8}{a^5 \times b^5} = a^2 \times b^3$$

$$b = 100 = 10^2 \text{ و } a = \frac{1}{10} = 10^{-1} (2)$$

$$C = (10^{-1})^2 \times (10^2)^3 = 10^{-2} \times 10^6 = 10^4$$

**تمرين 17:** أحسب وبسط حيث  $x \in \mathbb{R}$

$$B = [(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})]^2 \quad A = (\sqrt{5} + \sqrt{2})^2 - (\sqrt{5} - \sqrt{2})^2$$

$$E = (x+2)(x^2 - 2x + 4) \quad D = (3x-2)^3 \quad C = (\sqrt{2}+1)^3$$

$$F = (200520052006)^2 - (200520052005 \times 200520052007)$$

عندما تعجز الآلة الحاسبة

**الجواب:**

$$A = (\sqrt{5} + \sqrt{2})^2 - (\sqrt{5} - \sqrt{2})^2 = (\sqrt{5})^2 + 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2 - ((\sqrt{5})^2 - 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2)$$

$$A = 5 + 2\sqrt{10} + 2 - (5 - 2\sqrt{10} + 2) = 5 + 2\sqrt{10} + 2 - 5 + 2\sqrt{10} - 2 = 4\sqrt{10}$$

$$B = [(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})]^2 = ((\sqrt{2})^2 - (\sqrt{3})^2)^2 = (2 - 3)^2 = (-1)^2 = 1$$

$$C = (\sqrt{2} + 1)^3 = (\sqrt{2})^3 + 3(\sqrt{2})^2 \times 1 + 3\sqrt{2}(1)^2 + (1)^3 = 2\sqrt{2} + 3 \times 2 + 3\sqrt{2} + 1$$

$$C = 5\sqrt{2} + 7$$

$$D = (3x - 2)^3 = (3x)^3 - 3(3x)^2 \times 2 + 3 \times 3x \times (2)^2 - (2)^3$$

$$\left(\sqrt{a+\sqrt{a^2-b^2}}\right)^2 = \left(\frac{\sqrt{2}}{2}(\sqrt{a-b}+\sqrt{a+b})\right)^2 \text{ إذن وجدنا:}$$

$$\sqrt{a+\sqrt{a^2-b^2}} = \frac{\sqrt{2}}{2}(\sqrt{a-b}+\sqrt{a+b}) \text{ ومنه:}$$

**تمرين 22:** عمل التعبيرات التالية:  $a \in \mathbb{R}$  و  $b \in \mathbb{R}$  و  $x \in \mathbb{R}$

$$B=16-25x^2 \quad A=16x^2-8x+1$$

$$E=27+x^3 \quad D=(2x-1)^3-8 \quad C=1-(1-3x)^2$$

$$G=x^5+x^3-x^2-1 \quad F=x^2-2x^6+1$$

$$M=x^4-49 \quad H=x^3+1+2(x^2-1)-(x+1)$$

$$N=a^2+b^2-x^2+2ab$$

$$L=4x^2-4x\sqrt{5}+5+(1-2x)(2x-\sqrt{5})$$

$$R=x^2-6x+8 \quad \text{و} \quad K=(x-2)(3x-4)+x^3-8$$



التعميل هو كتابة مجموع على شكل جذاء

$$A=16x^2-8x+1=(4x)^2-2 \times 4x \times 1+1^2=(4x-1)^2 \text{ **الجواب:**}$$

$$B=16-25x^2=(4)^2-(5x)^2=(4-5x)(4+5x)$$

$$C=1-(1-3x)^2=1^2-(1-3x)^2=(1-(1-3x))(1+(1-3x))$$

$$C=(1-1+3x)(1+1-3x)=3x(2-3x)$$

$$D=(2x-1)^3-8=(2x-1)^3-2^3=$$

$$D=((2x-1)-2)((2x-1)^2+(2x-1) \times 2+2^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2) \text{ حسب المتطابقة التالية:}$$

$$D=(2x-3)((2x)^2-4x+1+4x-2+4)=(2x-3)(4x^2+3)$$

$$E=27+x^3=3^3+x^3=(3+x)(3^2-3x+x^2)$$

$$E=(3+x)(9-3x+x^2)$$

$$F=(x^6)^2-2x^6+1=(x^6)^2-2x^6 \times 1+1^2=(x^6-1)^2$$

$$G=x^5+x^3-x^2-1=x^3(x^2+1)-(x^2+1)=(x^3-1)(x^2+1)$$

$$H=x^3+1+2(x^2-1)-(x+1)=x^3+1^3+2(x^2-1^2)-(x+1)$$

$$H=(x+1)(x^2-x+1^2)+2(x+1)(x-1)-(x+1)$$

$$H=(x+1)(x^2-x+1+2(x-1)-1)=(x+1)(x^2-x+1+2x-2-1)$$

$$H=(x+1)(x^2+x-2)$$

$$M=x^4-49=x^4-(\sqrt{7})^4=(x^2)^2-(\sqrt{7}^2)^2$$

$$M=(x^2-\sqrt{7}^2)(x^2+\sqrt{7}^2)=(x-\sqrt{7})(x+\sqrt{7})(x^2+7)$$

$$N=a^2+b^2-x^2+2ab=a^2+2ab+b^2-x^2=(a+b)^2-x^2$$

$$N=(a+b-x)(a+b+x)$$

$$L=4x^2-4x\sqrt{5}+5+(1-2x)(2x-\sqrt{5})$$

$$L=(2x)^2-2 \times 2x \sqrt{5}+(\sqrt{5})^2+(1-2x)(2x-\sqrt{5})$$

$$L=(2x-\sqrt{5})^2+(1-2x)(2x-\sqrt{5})=(2x-\sqrt{5})(2x-\sqrt{5}+1-2x)$$

$$L=(2x-\sqrt{5})(1-\sqrt{5})$$

$$M=(x^2-2x)^2+2(x^2-2x) \times 1+1^2$$

$$M=(x^2)^2-2x^2 \times 2x+4x^2+2x^2-4x+1$$

$$M=x^4-4x^3+6x^2-4x+1$$

$$N=(x\sqrt{2}+\sqrt{5})(\sqrt{5}-x\sqrt{2})=(\sqrt{5}+x\sqrt{2})(\sqrt{5}-x\sqrt{2})$$

$$N=(\sqrt{5})^2-(x\sqrt{2})^2=5-2x^2$$

$$R=\left(x^3+\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}-x^3\right)=\left(\frac{\sqrt{3}}{2}+x^3\right)\left(\frac{\sqrt{3}}{2}-x^3\right)$$

$$R=\left(\frac{\sqrt{3}}{2}\right)^2-(x^3)^2=\frac{3}{4}-x^6$$

$$L=(3x+\sqrt{2}-\sqrt{5})(3x+\sqrt{2}+\sqrt{5})=((3x+\sqrt{2})-\sqrt{5})((3x+\sqrt{2})+\sqrt{5})$$

$$L=(3x)^2+6x\sqrt{2}+2-5=9x^2+6x\sqrt{2}-3$$

**تمرين 19:** أتمم الفراغات التالية :

$$10-4\sqrt{6}=(\dots-\dots)^2 \quad \text{و} \quad 4+2\sqrt{2}=(\dots+\dots)^2$$

**الجواب:**

$$4+2\sqrt{3}=4+2 \times \sqrt{3} \times 1=3+2 \times \sqrt{3} \times 1+1=(\sqrt{3})^2+2 \times \sqrt{3} \times 1+(1)^2$$

$$4+2\sqrt{3}=(\sqrt{3}+1)^2$$

$$10-4\sqrt{6}=10-2 \times 2 \times \sqrt{6}=(2)^2+2 \times \sqrt{6} \times 2+(\sqrt{6})^2$$

$$10-4\sqrt{6}=(2-\sqrt{6})^2$$

**تمرين 20:** أكتب التعبيرات التالية على شكل  $(a-b)^2$  أو  $(a+b)^2$  :

$$3-2\sqrt{2} \quad (4 \quad 9-4\sqrt{5} \quad (3 \quad 6+4\sqrt{2} \quad (2 \quad 11+6\sqrt{2} \quad (1$$

$$7-4\sqrt{3} \quad (6 \quad 12-6\sqrt{3} \quad (5$$

**الجواب:**

$$11+6\sqrt{2}=9+2 \times 3\sqrt{2}+2=3^2+2 \times 3\sqrt{2}+(\sqrt{2})^2=(3+\sqrt{2})^2 \quad (1$$

$$6+4\sqrt{2}=4+2 \times 2\sqrt{2}+2=2^2+2 \times 2\sqrt{2}+(\sqrt{2})^2=(2+\sqrt{2})^2 \quad (2$$

$$9-4\sqrt{5}=4-2 \times 2\sqrt{5}+5=2^2-2 \times 2\sqrt{5}+(\sqrt{5})^2=(2-\sqrt{5})^2 \quad (3$$

$$3-2\sqrt{2}=1-2 \times 1\sqrt{2}+2=1^2-2 \times 1\sqrt{2}+(\sqrt{2})^2=(1-\sqrt{2})^2 \quad (4$$

$$12-6\sqrt{3}=9-2 \times 3\sqrt{3}+3=3^2-2 \times 3\sqrt{3}+(\sqrt{3})^2=(3-\sqrt{3})^2 \quad (5$$

$$7-4\sqrt{3}=4-2 \times 2\sqrt{3}+3=2^2-2 \times 2\sqrt{3}+(\sqrt{3})^2=(2-\sqrt{3})^2 \quad (6$$

**تمرين 21:**  $a \geq b$  و  $b \in \mathbb{R}^*$   $b \in \mathbb{R}^*$   $a \in \mathbb{R}^*$

$$\sqrt{a+\sqrt{a^2-b^2}} = \frac{\sqrt{2}}{2}(\sqrt{a-b}+\sqrt{a+b}) \text{ بين أن:}$$

**الجواب:**

لكي نبين أن عددين موجبين متساويين يكفي أن نبين أن مربعيهما متساويين

$$\left(\sqrt{a+\sqrt{a^2-b^2}}\right)^2 = a + \sqrt{a^2-b^2}$$

$$\left(\frac{\sqrt{2}}{2}(\sqrt{a-b}+\sqrt{a+b})\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 \times (\sqrt{a-b}+\sqrt{a+b})^2$$

$$= \frac{2}{4} \times (\sqrt{a-b}+\sqrt{a+b})^2 = \frac{1}{2} \times ((\sqrt{a-b})^2 + 2\sqrt{a-b}\sqrt{a+b} + (\sqrt{a+b})^2)$$

$$= \frac{2}{4} \times (\sqrt{a-b}+\sqrt{a+b})^2 = \frac{1}{2} \times (a-b+2\sqrt{(a-b)(a+b)}+a+b)$$

$$= \frac{1}{2} \times (2a+2\sqrt{(a-b)(a+b)}) = a + \sqrt{(a-b)(a+b)} = a + \sqrt{a^2-b^2}$$

مجموع عددين موجبين  $a$  و  $b$  اذن هو عدد موجب ومنه :

$$u = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$$

$$v^2 = 36 \text{ يعني } v = \sqrt{36} \text{ أو } v = -\sqrt{36} \text{ ولكن نعلم أن : } v = a - b$$

ونلاحظ أن :  $a > b$  اذن هو عدد موجب ومنه :  $v = \sqrt{36} = 6$

(4) استنتاج كتابة للعددين  $a$  و  $b$  :

$$\text{وجدنا : } \begin{cases} u = 2\sqrt{10} \\ v = 6 \end{cases} \text{ يعني } \begin{cases} a + b = 2\sqrt{10} \\ a - b = 6 \end{cases} \text{ وجمع المعادلتين طرف}$$

$$\text{لطرف نجد : } 2a = 6 + 2\sqrt{10} \text{ أي } a = \frac{6 + 2\sqrt{10}}{2} = 3 + \sqrt{10}$$

$$\text{ولدينا : } a + b = 2\sqrt{10} \text{ يعني } b = 2\sqrt{10} - a \text{ يعني } b = 2\sqrt{10} - 3 - \sqrt{10}$$

$$\text{يعني } b = \sqrt{10} - 3$$

**تمرين 26:** (1) بين أن :  $\sqrt{3 + \sqrt{5}} + \sqrt{3 - \sqrt{5}} = \sqrt{10}$

(2) بين أن :  $\sqrt{\frac{6 + \sqrt{31}}{2}} + \sqrt{\frac{6 - \sqrt{31}}{2}} = \sqrt{6 + \sqrt{5}}$

(3) بين أن :  $\sqrt{9 - \sqrt{79}} + \sqrt{9 + \sqrt{79}} = \sqrt{18 + \sqrt{8}}$

**الجواب:** (1)  $\sqrt{3 + \sqrt{5}} + \sqrt{3 - \sqrt{5}} = \sqrt{10}$  ؟

نضع :  $B = \sqrt{3 + \sqrt{5}} + \sqrt{3 - \sqrt{5}}$

ونحسب  $B^2$  :  $B^2 = (\sqrt{3 + \sqrt{5}})^2 + 2\sqrt{3 + \sqrt{5}}\sqrt{3 - \sqrt{5}} + (\sqrt{3 - \sqrt{5}})^2$

$$B^2 = 3 + \sqrt{5} + 2\sqrt{(3 + \sqrt{5})(3 - \sqrt{5})} + 3 - \sqrt{5}$$

$$B^2 = 6 + 2\sqrt{3^2 - (\sqrt{5})^2} = 6 + 2\sqrt{9 - 5} = 6 + 2\sqrt{4} = 6 + 4 = 10$$

$$B^2 = 10 \text{ يعني } B = \sqrt{10} \text{ أو } B = -\sqrt{10}$$

ونعلم أن  $B > 0$  اذن  $B = \sqrt{10}$

(2) نضع :  $B = \sqrt{\frac{6 + \sqrt{31}}{2}} + \sqrt{\frac{6 - \sqrt{31}}{2}}$

ونحسب  $B^2$  :  $B^2 = \left(\sqrt{\frac{6 + \sqrt{31}}{2}}\right)^2 + 2\sqrt{\frac{6 + \sqrt{31}}{2}}\sqrt{\frac{6 - \sqrt{31}}{2}} + \left(\sqrt{\frac{6 - \sqrt{31}}{2}}\right)^2$

$$B^2 = \frac{6 + \sqrt{31}}{2} + 2\sqrt{\left(\frac{6 + \sqrt{31}}{2}\right)\left(\frac{6 - \sqrt{31}}{2}\right)} + \frac{6 - \sqrt{31}}{2}$$

$$B^2 = 6 + 2\sqrt{\frac{36 - 1}{4}} = 6 + 2\sqrt{\frac{5}{4}} = 6 + \sqrt{5}$$

$$B^2 = 6 + \sqrt{5} \text{ يعني } B = \sqrt{6 + \sqrt{5}} \text{ أو } B = -\sqrt{6 + \sqrt{5}}$$

ونعلم أن  $B > 0$  اذن  $B = \sqrt{6 + \sqrt{5}}$

ومنه  $\sqrt{\frac{6 + \sqrt{31}}{2}} + \sqrt{\frac{6 - \sqrt{31}}{2}} = \sqrt{6 + \sqrt{5}}$

(3)  $\sqrt{9 - \sqrt{79}} + \sqrt{9 + \sqrt{79}} = \sqrt{18 + \sqrt{8}}$  ؟

نضع :  $B = \sqrt{9 - \sqrt{79}} + \sqrt{9 + \sqrt{79}}$

ونحسب  $B^2$  :  $B^2 = (\sqrt{9 - \sqrt{79}})^2 + 2\sqrt{9 - \sqrt{79}}\sqrt{9 + \sqrt{79}} + (\sqrt{9 + \sqrt{79}})^2$

$$B^2 = 9 - \sqrt{79} + 2\sqrt{(9 - \sqrt{79})(9 + \sqrt{79})} + 9 + \sqrt{79}$$

$$B^2 = 18 + 2\sqrt{81 - 79} = 18 + \sqrt{8}$$

$$B^2 = 18 + \sqrt{8} \text{ يعني } B = \sqrt{18 + \sqrt{8}} \text{ أو } B = -\sqrt{18 + \sqrt{8}}$$

ونعلم أن  $B > 0$  اذن  $B = \sqrt{18 + \sqrt{8}}$

ومنه  $\sqrt{9 - \sqrt{79}} + \sqrt{9 + \sqrt{79}} = \sqrt{18 + \sqrt{8}}$

« c'est en forgeant que l'on devient forgeron » dit un proverbe.  
c'est en s'entraînant régulièrement aux calculs et exercices  
que l'on devient un mathématicien



$$K = (x - 2)(3x - 4) + x^3 - 8$$

$$K = (x - 2)(3x - 4) + (x - 2)(x^2 + 2x + 4)$$

$$K = (x - 2)(3x - 4 + x^2 + 2x + 4) = (x - 2)(x^2 + 5x)$$

$$K = x(x - 2)(x + 5)$$

$$R = x^2 - 6x + 8 = x^2 - 6x + 9 - 1 = x^2 - 2 \times 3x + 3^2 - 1$$

$$R = (x - 3)^2 - 1^2 = (x - 3 - 1)(x - 3 + 1) = (x - 4)(x - 2)$$

**تمرين 23:** نضع :  $a \in \mathbb{R}$  :  $A = (a + 1)^2 - (a - 1)^2$

(1) أحسب وبسط  $A$  (2) استنتج تبسيطا للعدد :  $(9999999)^2 - (9999997)^2$

**الجواب:** (1)  $A = (a + 1)^2 - (a - 1)^2 = a^2 + 2a + 1 - (a^2 - 2a + 1)$

$$A = a^2 + 2a + 1 - a^2 + 2a - 1 = 4a$$

(2) لنأخذ :  $a = 9999998$

$$A = (9999999)^2 - (9999997)^2 = 4 \times 9999998$$

ومنه :  $(9999999)^2 - (9999997)^2 = 39999992$

**تمرين 24:** نضع :  $B = \sqrt{6 - 2\sqrt{5}} - \sqrt{6 + 2\sqrt{5}}$

(1) حدد اشارة العدد  $B$

(2) أحسب  $B^2$  (3) استنتج كتابة مبسطة للعدد  $B$

**الجواب:** (1)  $B = \sqrt{6 - 2\sqrt{5}} - \sqrt{6 + 2\sqrt{5}}$

نلاحظ أن :  $6 - 2\sqrt{5} < 6 + 2\sqrt{5}$  ومنه  $\sqrt{6 - 2\sqrt{5}} < \sqrt{6 + 2\sqrt{5}}$

وبالتالي :  $B < 0$

(2)  $B^2 = (\sqrt{6 - 2\sqrt{5}} - \sqrt{6 + 2\sqrt{5}})^2$

$$B^2 = (\sqrt{6 - 2\sqrt{5}})^2 - 2\sqrt{6 - 2\sqrt{5}}\sqrt{6 + 2\sqrt{5}} + (\sqrt{6 + 2\sqrt{5}})^2$$

$$B^2 = 6 - 2\sqrt{5} - 2\sqrt{(6 - 2\sqrt{5})(6 + 2\sqrt{5})} + 6 + 2\sqrt{5}$$

$$B^2 = 12 - 2\sqrt{6^2 - (2\sqrt{5})^2} = 12 - 2\sqrt{36 - 20} = 12 - 2\sqrt{16}$$

$$B^2 = 12 - 2 \times 4 = 4$$

(3)  $B^2 = 4$  يعني  $B = \sqrt{4}$  أو  $B = -\sqrt{4}$

يعني  $B = 2$  أو  $B = -2$  ونعلم أن  $B < 0$  اذن  $B = -2$

**تمرين 25:** نضع :  $a = \sqrt{19 + 6\sqrt{10}}$  و  $b = \sqrt{19 - 6\sqrt{10}}$

(1) بين أن :  $a \times b = 1$

(2) نضع :  $u = a + b$  و  $v = a - b$

أحسب  $u^2$  و  $v^2$

(3) استنتج كتابة للعددين  $u$  و  $v$

(4) استنتج كتابة للعددين  $a$  و  $b$

**الجواب:** (1)

$$ab = \sqrt{19 + 6\sqrt{10}}\sqrt{19 - 6\sqrt{10}} = \sqrt{(19 + 6\sqrt{10})(19 - 6\sqrt{10})}$$

$$ab = \sqrt{19^2 - (6\sqrt{10})^2} = \sqrt{361 - 360} = \sqrt{1} = 1$$

(2)  $u = a + b$  و  $v = a - b$

$$u^2 = (a + b)^2 = a^2 + b^2 + 2ab = a^2 + b^2 + 2 \times 1$$

$$u^2 = 19 + 6\sqrt{10} + 19 - 6\sqrt{10} + 2 \times 1 = 40$$

$$v^2 = (a - b)^2 = a^2 + b^2 - 2ab = a^2 + b^2 - 2 \times 1$$

$$v^2 = 19 + 6\sqrt{10} + 19 - 6\sqrt{10} - 2 \times 1 = 36$$

(3) استنتج كتابة للعددين  $u$  و  $v$  :

$$u^2 = 40 \text{ يعني } u = \sqrt{40} \text{ أو } u = -\sqrt{40}$$

ولكن نعلم أن :  $u = a + b$