

تمارين وحلولها

$$\frac{\pi}{5} = \frac{200}{12} = 40 \text{ gr}$$

$$\frac{\pi}{6} = \frac{180^\circ}{6} = 30^\circ \quad * \text{ لدينا :}$$

$$\frac{\pi}{6} = \frac{200}{6} = 33,33 \text{ gr}$$

إذن

$\frac{\pi}{6}$	$\frac{\pi}{5}$	$\frac{\pi}{4}$	$\frac{\pi}{12}$	$\frac{\pi}{8}$	الرديان
30°	36°	45°	15°	22,5°	الدرجة
33,33gr	40°	50	16,66gr	25 gr	الغراد

تمرين 2 :

1 - احسب بالرديان قياسات زوايا مثلث متساوي الأضلاع.

2 - احسب بالراديان قياسات زوايا مثلث قائم الزاوية ومتساوي الساقين.

الجواب :

1 - ليكن ABC مثلثا متساوي الأضلاع.

$$\hat{A} = \hat{B} = \hat{C} = 60^\circ \text{ نعلم أن}$$

$$\hat{A} = \hat{B} = \hat{C} = \frac{\pi}{3} \text{ إذن :}$$

2 - ليكن EFG مثلثا متساوي الساقين وقائم

الزاوية مثلا في E.

$$\hat{E} = 90^\circ \text{ نعلم أن :}$$

تمرين 1 :

أتمم الجدول التالي :

$\frac{\pi}{6}$				$\frac{\pi}{8}$	الرديان
	36°		15°		الدرجة
		50			الغراد

الجواب :

$$\frac{\pi}{8} = \frac{180}{8} = 22,5^\circ \text{ لدينا *}$$

$$\frac{\pi}{8} = \frac{200}{8} = 25 \text{ gr}$$

$$\frac{x}{\pi} = \frac{15}{180} \text{ لدينا *}$$

$$x = \frac{15\pi}{180} \text{ أي أن}$$

$$x = \frac{\pi}{12}$$

$$\frac{\pi}{12} = \frac{200}{12} \approx 16,66 \text{ gr لدينا *}$$

$$50 \text{ gr} = \frac{200}{4}x = \frac{180^\circ}{4} = 45^\circ$$

$$50 \text{ gr} = \frac{\pi}{4}$$

$$\frac{x}{\pi} = \frac{36^\circ}{180}$$

$$\frac{x}{\pi} = \frac{1}{5}$$

$$x = \frac{\pi}{5}$$

* لدينا

$$= 100\pi + \frac{4\pi}{5}$$

$$\frac{504\pi}{5} \equiv \frac{4\pi}{5} [2\pi] \quad \text{إذن}$$

ومنه الأفضول المنحني الرئيسي لـ M هو $\frac{4\pi}{5}$

$$\frac{4\pi}{5} \in]-\pi, \pi[\quad \text{لأن}$$

$$\text{ج - لدينا : } -\frac{277\pi}{4} = \frac{-280\pi + 3\pi}{4}$$

$$= -70\pi + \frac{3\pi}{4}$$

$$-\frac{277\pi}{4} \equiv \frac{3\pi}{4} [2\pi] \quad \text{إذن}$$

إذن الأفضول المنحني الرئيسي للنقطة M هو $\frac{3\pi}{4}$

$$\frac{3\pi}{4} \in]-\pi, \pi[\quad \text{لأن}$$

$$\text{2 - أ - لدينا : } x = \frac{45\pi}{4}$$

$$= \frac{48\pi - 3\pi}{4}$$

$$= 12\pi - \frac{3\pi}{4}$$

$$x = 12\pi + y \quad \text{إذن}$$

$$\text{ومنه } x \equiv y [2\pi]$$

إذن x و y أفضولان منحنيان لنفس النقطة

ب - نفترض أن : $x \equiv y [2\pi]$ أي أنه يوجد k

$$\text{من } \mathbb{Z} \text{ حيث : } x = y + 2k\pi$$

$$\text{أي أن } -\frac{123\pi}{5} = \frac{337\pi}{5} + 2k\pi$$

$$\text{أي أن } -\frac{123\pi}{5} - \frac{337\pi}{5} = 2k\pi$$

$$\text{ومنه } 2k\pi = -\frac{500\pi}{5}$$

$$\hat{F} = \hat{G} = 45^\circ \quad \text{و}$$

$$\hat{E} = \frac{\pi}{2} \quad \text{إذن}$$

$$\hat{F} = \hat{G} = \frac{\pi}{4} \quad \text{و}$$

تمرين 3 :

1 - حدد الأفضول المنحني الرئيسي للنقطة في

الحالات التالية :

$$\text{أ - } M\left(-\frac{99\pi}{7}\right)$$

$$\text{ب - } M\left(\frac{504\pi}{7}\right)$$

$$\text{ج - } M\left(-\frac{277\pi}{4}\right)$$

2 - هل العدان x و y هما أفضولان منحنيان

لنفس النقطة على الدائرة المثلثية في الحالتين

التاليتين :

$$\text{أ - } x = -\frac{3\pi}{4} \quad \text{و} \quad x = \frac{45\pi}{4}$$

$$\text{ب - } x = \frac{337\pi}{5} \quad \text{و} \quad x = -\frac{123\pi}{5}$$

الجواب :

$$\text{1 - أ - لدينا } -\frac{99\pi}{7} = -\frac{98\pi + \pi}{7}$$

$$= -14\pi - \frac{\pi}{7}$$

$$\text{إذن : } -\frac{99\pi}{7} \equiv -\frac{\pi}{7} [2\pi]$$

$$\text{و } -\frac{\pi}{7} \in]-\pi, \pi[$$

إذن الأفضول المنحني الرئيسي لـ M هو $-\frac{\pi}{7}$

$$\text{ب - لدينا } \frac{504\pi}{5} = \frac{500\pi + 4\pi}{5}$$

$$\frac{41\pi}{6} \equiv \frac{5\pi}{6} [2\pi] \quad \text{إذن}$$

الأفصول المنحني الرئيسي لـ C هو $\frac{5\pi}{6}$.

$$* \text{ نعتبر } D \left(\frac{25\pi}{4} \right)$$

$$\frac{25\pi}{4} \equiv \frac{24\pi + \pi}{4} \quad \text{لدينا}$$

$$= 6\pi + \frac{\pi}{4}$$

$$\frac{25\pi}{4} \equiv \frac{\pi}{4} [2\pi] \quad \text{إذن}$$

الأفصول المنحني الرئيسي لـ D هو $\frac{\pi}{4}$.

$$* \text{ نعتبر } B \left(-\frac{33\pi}{4} \right)$$

$$\frac{33\pi}{4} \equiv \frac{-32\pi - \pi}{4} \quad \text{لدينا}$$

$$= -8\pi - \frac{\pi}{4}$$

$$\frac{33\pi}{4} \equiv -\frac{\pi}{4} [2\pi] \quad \text{إذن}$$

الأفصول المنحني الرئيسي لـ E هو $\frac{\pi}{4}$.

$$* \text{ نعتبر } F \left(\frac{169\pi}{3} \right)$$

$$\frac{169\pi}{3} \equiv \frac{168\pi + \pi}{3} \quad \text{لدينا}$$

$$= 56\pi + \frac{\pi}{3}$$

$$= 56\pi + \pi + \frac{\pi}{3}$$

$$\frac{169\pi}{3} \equiv \pi + \frac{\pi}{3} [2\pi] \quad \text{إذن}$$

$$\equiv -\pi + \frac{\pi}{3} [2\pi]$$

$$\equiv -\frac{2\pi}{3} [2\pi]$$

الأفصول المنحني الرئيسي لـ E هو $-\frac{2\pi}{3}$.

$$2k\pi = -100\pi \quad \text{ومنه}$$

$$2k\pi = -100$$

$$2k = -100$$

$$k = -50 \quad \text{ومنه}$$

وبما أن $k \in \mathbb{Z}$ فإن $x \equiv y [2\pi]$ إذن x و y

أفصولان منحنيان لنفس النقطة.

تمرين 4 :

مثل على دائرة مثلثية النقط ذات الأفاصل المنحنية التالية :

$$\frac{169\pi}{4}; -\frac{33\pi}{4}; \frac{25\pi}{4}; \frac{41\pi}{6}; \frac{8\pi}{3}; -\pi$$

الجواب :

* نعتبر $A'(-\pi)$ إذن الأفصول المنحني الرئيسي

لـ A' هو : π

$$* \text{ نعتبر } B \left(\frac{8\pi}{3} \right)$$

$$\frac{8\pi}{3} \equiv \frac{6\pi + 2\pi}{3} = 2\pi + \frac{2\pi}{3} \quad \text{لدينا}$$

$$\frac{8\pi}{3} \equiv \frac{2\pi}{3} [2\pi] \quad \text{إذن}$$

الأفصول المنحني الرئيسي لـ B هو $\frac{2\pi}{3}$.

$$* \text{ نعتبر } C \left(\frac{41\pi}{6} \right)$$

$$\frac{41\pi}{6} \equiv \frac{36\pi + 5\pi}{6} \quad \text{لدينا}$$

$$= 6\pi + \frac{5\pi}{6}$$

$$(\widehat{\vec{CB}, \vec{BD}}) \equiv (-\widehat{\vec{BC}, \vec{BD}}) [2\pi] \quad \text{لدينا}$$

$$\equiv \pi + (\widehat{\vec{BC}, \vec{BD}}) [2\pi]$$

$$\equiv \pi + \frac{\pi}{3} [2\pi]$$

$$\equiv \frac{4\pi}{3} [2\pi]$$

$$\equiv \frac{6\pi - 2\pi}{3} [2\pi]$$

$$\equiv -\frac{2\pi}{3} [2\pi]$$

إذن القياس الرئيسي للزاوية $(\widehat{\vec{CB}, \vec{BD}})$

هو : $-\frac{2\pi}{3}$

لدينا :

$$(\widehat{\vec{BA}, \vec{BD}}) \equiv (\widehat{\vec{BA}, \vec{BC}}) + (\widehat{\vec{BC}, \vec{BD}}) [2\pi]$$

$$\equiv \frac{\pi}{4} + \frac{\pi}{3} [2\pi]$$

$$\equiv \frac{7\pi}{12} [2\pi]$$

إذن القياس الرئيسي للزاوية $(\widehat{\vec{BA}, \vec{BD}})$

هو : $\frac{7\pi}{12}$

تمرين 6 :

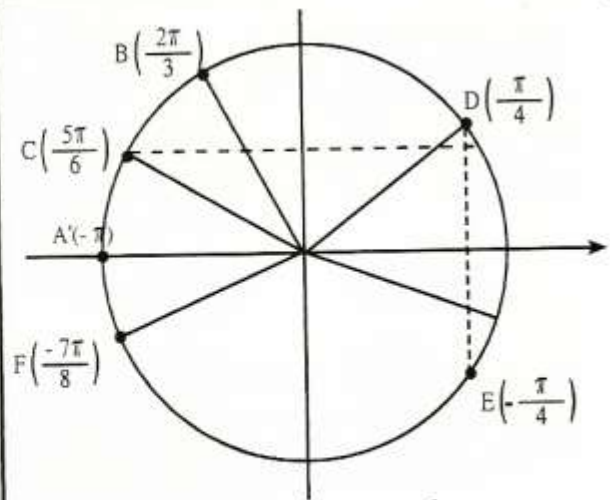
ABC مثلث بين أن :

$$(\widehat{\vec{AB}, \vec{AC}}) + (\widehat{\vec{CA}, \vec{CB}}) + (\widehat{\vec{BC}, \vec{BA}}) \equiv \pi [2\pi]$$

الجواب :

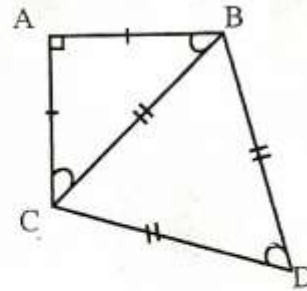
$$* \text{ لدينا } (\widehat{\vec{BC}, \vec{BA}}) + (\widehat{\vec{CA}, \vec{CB}}) + (\widehat{\vec{AB}, \vec{AC}})$$

$$\equiv (\widehat{\vec{AB}, \vec{AC}}) + (-\widehat{\vec{AC}, \vec{BC}}) + (\widehat{\vec{BC}, \vec{BA}}) [2\pi]$$



تمرين 5 :

نعتبر الشكل التالي :



اعط القياس الرئيسي لكل من الزوايا التالية :

$$(\widehat{\vec{AB}, \vec{AC}}), (\widehat{\vec{DC}, \vec{DB}}), (\widehat{\vec{BA}, \vec{BC}})$$

$$(\widehat{\vec{BA}, \vec{BD}}), (\widehat{\vec{CB}, \vec{BD}})$$

الجواب :

$$* \text{ لدينا } (\widehat{\vec{BA}, \vec{BC}}) \equiv \frac{\pi}{4} [2\pi]$$

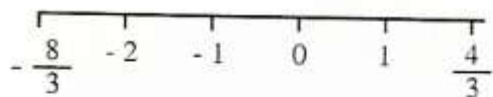
القياس الرئيسي للزاوية $(\widehat{\vec{BA}, \vec{BC}})$ هو $\frac{\pi}{4}$.

$$\text{لدينا : } (\widehat{\vec{DC}, \vec{DB}}) \equiv -\frac{\pi}{3} [2\pi]$$

إذن القياس الرئيسي للزاوية $(\widehat{\vec{DC}, \vec{DB}})$ هو $-\frac{\pi}{3}$.

$$* \text{ لدينا } (\widehat{\vec{AB}, \vec{AC}}) \equiv -\frac{\pi}{2} [2\pi]$$

إذن القياس الرئيسي $(\widehat{\vec{AB}, \vec{AC}})$ هو $-\frac{\pi}{2}$.



بما أن $k \in \mathbb{Z}$ فإن k تأخذ القيم $1, 0, -1, -2$

$$M_0 \left(\frac{\pi}{3} \right) \quad \text{إذن } k = 0$$

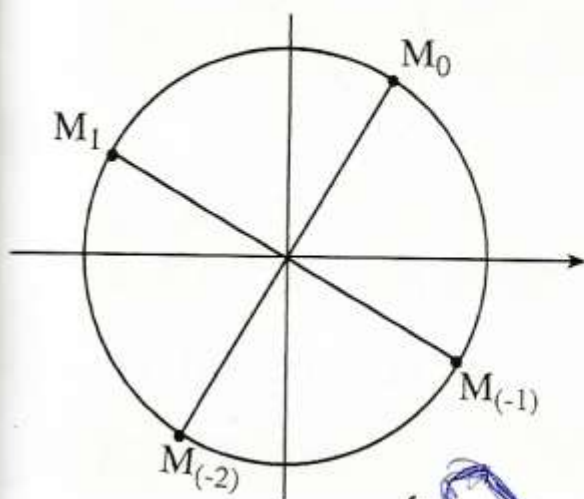
$$M_1 \left(\frac{5\pi}{6} \right) \quad \text{إذن } k = 1$$

$$M_{-1} \left(-\frac{\pi}{6} \right) \quad \text{إذن } k = -1$$

$$M_{-2} \left(-\frac{2\pi}{6} \right) \quad \text{إذن } k = -2$$

لتمثيل النقط M_k يكفي أن نتمثيل النقط .

M_{-2}, M_{-1}, M_1, M_0



تمرين 8 :

ليكن x من المجموعة \mathbb{R} بسط مايلي :

$$A(x) = 2\sin^2(x) + 3\cos^2(x) - 1$$

$$B(x) = (\cos x + \sin x)^2 - 1$$

$$C(x) = \cos^2 x - \cos^2 x \cdot \sin^2 x$$

$$D(x) = (2\cos x + \sin x)^2 + (\cos x - 2\sin x)^2$$

$$E(x) = \cos^5 x + \cos^3 x \cdot \sin^2 x$$

$$F(x) = \cos^4 x - \cos^2 x + \sin^2 x - \sin^4 x$$

$$G(x) = \cos^6 x + \sin^6 x + 3\sin^2 x \cdot \cos^2 x$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{AC}) + (\overrightarrow{AC}, \overrightarrow{BC}) + (\overrightarrow{BC}, \overrightarrow{BA}) [2\pi]$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{BC}) + (\overrightarrow{BC}, \overrightarrow{BA}) [2\pi]$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{BA}) [2\pi]$$

$$\equiv (\overrightarrow{AB}, \overrightarrow{AB}) [2\pi]$$

$$\equiv \pi + (\overrightarrow{AB}, \overrightarrow{AB}) [2\pi]$$

$$\equiv \pi + 0 [2\pi]$$

إذن :

$$(\overrightarrow{AB}, \overrightarrow{AC}) + (\overrightarrow{CA}, \overrightarrow{CB}) + (\overrightarrow{BC}, \overrightarrow{BA}) \equiv \pi [2\pi]$$

تمرين 7 :

مثل على دائرة مثلثية النقط M_k التي أفصلها المنحنية هي الأعداد :

$$k \in \mathbb{Z}, \frac{\pi}{3} + \frac{k\pi}{2}$$

الجواب :

$$* \text{ لدينا } B \left(\frac{\pi}{3} + \frac{k\pi}{2} \right) \quad k \in \mathbb{Z}$$

لنحدد قيم k التي من أجلها يكون $\frac{\pi}{3} + \frac{k\pi}{2}$ قياسا رئيسيا لـ M_k .

$$\text{إذن } -\pi < \frac{k\pi}{2} + \frac{\pi}{3} \leq \pi$$

$$\text{أي أن } -1 < \frac{1}{3} + \frac{k}{2} \leq 1$$

$$\text{إذن } -\frac{4}{3} < \frac{k}{2} \leq \frac{2}{3}$$

$$\text{أي أن } -\frac{8}{3} < k \leq \frac{4}{3}$$

$$C(x) = \cos^3 x$$

إذن :

* لدينا

$$\begin{aligned} F(x) &= \cos^4 x - \cos^2 x + \sin^2 x - \sin^4 x \\ &= \cos^2 x (\cos^2 x - 1) + \sin^2 x (1 - \sin^2 x) \\ &= \cos^2 x \cdot (-\sin^2 x) + \sin^2 x \cdot \cos^2 x \\ &= -\cos^2 x \cdot \sin^2 x + \sin^2 x \cdot \cos^2 x \\ &= 0 \end{aligned}$$

$$F(x) = 0$$

إذن :

* لدينا

$$\begin{aligned} G(x) &= \cos^6 x + \sin^6 x + 3\sin^2 x \cdot \cos^2 x \\ &= (\cos^2 x)^3 + (\sin^2 x)^3 + 3\sin^2 x \cdot \cos^2 x \\ &= (\cos^2 x + \sin^2 x) (\cos^4 x - \cos^2 x \cdot \sin^2 x + \sin^4 x) + 3\sin^2 x \cdot \cos^2 x \\ &= \cos^4 x - \cos^2 x \cdot \sin^2 x + \sin^4 x + 3\sin^2 x \cdot \cos^2 x \\ &= \cos^4 x + \sin^4 x + 2\sin^2 x \cdot \cos^2 x \\ &= (\cos^2 x)^2 + (\sin^2 x)^2 + 2\sin^2 x \cdot \cos^2 x \\ &= (\cos^2 x + \sin^2 x)^2 \\ &= 1^2 \\ &= 1 \end{aligned}$$

$$F(x) = 1$$

إذن :

تمارين 9 :

1 - ليكن x من المجال $\left[0, \frac{\pi}{2}\right]$ و $\sin x = \frac{\sqrt{5}}{4}$ احسب $\cos x$ و $\tan x$.

2 - إذا علمت أن $x \in \left[\frac{\pi}{2}, \pi\right]$

الجواب :

$$\begin{aligned} A(x) &= 2\sin^2(x) + 3\cos^2(x) - 1 \quad \text{لدينا}^* \\ &= 2\sin^2 x + 3(1 - \sin^2 x) - 1 \\ &= 2\sin^2 x + 3 - 3\sin^2 x - 1 \\ &= 2 - \sin^2 x \end{aligned}$$

$$A(x) = 2 - \sin^2 x$$

إذن :

$$\begin{aligned} B(x) &= (\cos x + \sin x)^2 - 1 \quad \text{لدينا}^* \\ &= \cos^2 x + \sin^2 x + 2\sin x \cos x - 1 \\ &= 1 + 2\sin x \cdot \cos x - 1 \\ &= 2\sin x \cos x \end{aligned}$$

$$B(x) = 2\sin x \cdot \cos x$$

إذن :

$$\begin{aligned} C(x) &= \cos^2 - \cos^2 x \cdot \sin^2 x \quad \text{لدينا}^* \\ &= \cos^2 x (1 - \sin^2 x) \\ &= \cos^2 x \cdot \cos^2 x \end{aligned}$$

$$C(x) = \cos^4 x$$

إذن :

* لدينا

$$\begin{aligned} D(x) &= (2\cos x + \sin x)^2 + (\cos x - 2\sin x)^2 \\ &= 4\cos^2 x + \sin^2 x + 4\cos x \cdot \sin x + \cos^2 x \\ &\quad + 4\sin^2 x - 4\sin x \cos x \\ &= 5\cos^2 x + 5\sin^2 x \\ &= 5(\cos^2 x + \sin^2 x) \\ &= 5 \times 1 \end{aligned}$$

$$D(x) = 5$$

إذن :

$$\begin{aligned} E(x) &= \cos^5 x + \cos^3 x \cdot \sin^2 x \quad \text{لدينا}^* \\ &= \cos^3 (\cos^2 x + \sin^2 x) \\ &= \cos^3 x \cdot 1 \end{aligned}$$

$$\sin^2 x + \left(-\frac{2}{3}\right)^2 = 1 \quad \text{أي أن}$$

$$\sin^2 x = 1 - \frac{4}{9}$$

$$\sin^2 x = \frac{5}{9}$$

$$\sin x = -\frac{\sqrt{5}}{3} \quad \text{أو} \quad \sin x = \frac{\sqrt{5}}{3} \quad \text{ومنه}$$

$$\sin x = \frac{\sqrt{5}}{3} \quad \text{فإن} \quad \sin x \geq 0$$

$$\text{نعلم أن لكل } x \text{ من } \left] \frac{\pi}{2}, \pi \right]$$

$$\begin{aligned} \tan x &= \frac{\sin x}{\cos x} \\ &= \frac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = -\frac{\sqrt{5}}{2} \end{aligned}$$

$$\tan x = -\frac{\sqrt{5}}{2} \quad \text{إذن}$$

$$\tan \alpha = \sqrt{7} \quad \text{لدينا}$$

$$\cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha} \quad \text{نعلم أن}$$

$$= \frac{1}{1 + (\sqrt{7})^2}$$

$$= \frac{1}{8}$$

$$\cos \alpha = \sqrt{\frac{1}{8}} \quad \text{أو} \quad \cos \alpha = -\sqrt{\frac{1}{8}} \quad \text{إذن}$$

$$\cos \alpha = \frac{1}{2\sqrt{2}} \quad \text{أو} \quad \cos \alpha = \frac{-1}{2\sqrt{2}}$$

$$\cos \alpha = \frac{\sqrt{2}}{4} \quad \text{أو} \quad \cos \alpha = \frac{-\sqrt{2}}{4}$$

$$\cos \alpha = \frac{-\sqrt{2}}{4} \quad \text{فإن} \quad \cos \alpha \leq 0$$

$$\text{و } \cos x = -\frac{2}{3} \quad \text{فاحسب } \sin x \text{ و } \tan x$$

$$\alpha \in \left[-\pi, -\frac{\pi}{2}\right] \quad \text{إذا علمت أن}$$

$$\tan \alpha = \sqrt{7} \quad \text{و}$$

$$\text{فاحسب } \sin \alpha \text{ و } \cos \alpha$$

$$-4 \quad \text{إذا علمت أن } \cos \frac{5\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\text{فاحسب } \cos \frac{5\pi}{12} \quad \text{ثم } \tan \frac{5\pi}{12}$$

الجواب:

$$-1 \quad \text{لدينا } \sin x = \frac{\sqrt{5}}{4}$$

$$\text{نعلم أن } \cos^2 x + \sin^2 x = 1$$

$$\cos^2 x + \left(\frac{\sqrt{5}}{4}\right)^2 = 1 \quad \text{أي أن}$$

$$\cos^2 x = 1 - \frac{5}{16} \quad \text{إذن}$$

$$\cos^2 x = \frac{11}{16} \quad \text{أي أن}$$

$$\cos x = \frac{\sqrt{11}}{4} \quad \text{أو} \quad \cos x = -\frac{\sqrt{11}}{4} \quad \text{ومنه}$$

$$\text{وبما أن } \cos x \geq 0 \quad \text{فإن } x \in \left[0, \frac{\pi}{2}\right]$$

$$\cos x = \frac{\sqrt{11}}{4} \quad \text{ومنه}$$

$$\text{نعلم أن لكل } x \text{ من } \left[0, \frac{\pi}{2}\right]$$

$$\begin{aligned} \tan x &= \frac{\sin x}{\cos x} \\ &= \frac{\frac{\sqrt{5}}{4}}{\frac{\sqrt{11}}{4}} = \frac{\sqrt{5}}{\sqrt{11}} = \frac{\sqrt{55}}{11} \end{aligned}$$

$$-2 \quad \text{لدينا } \cos x = -\frac{2}{3}$$

$$\text{نعلم أن } \cos^2 x + \sin^2 x = 1$$

وبما $0 < \frac{5\pi}{12} < \frac{\pi}{2}$ فإن $\sin \frac{5\pi}{12} > 0$

ومنه $\sin \frac{5\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4}$

لدينا $\tan \frac{5\pi}{12} = \frac{\sin \frac{5\pi}{12}}{\cos \frac{5\pi}{12}}$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}$$

$$= \frac{(\sqrt{6} + \sqrt{2})^2}{6 - 2}$$

$$= \frac{8 + 2\sqrt{12}}{4}$$

$$= \frac{8 + 4\sqrt{3}}{4}$$

$$= \frac{4(2 + \sqrt{3})}{4}$$

إذن : $\tan \frac{5\pi}{12} = 2 + \sqrt{3}$

نعلم أن $\tan \alpha = \frac{\sin x}{\cos x}$

إذن $\sin \alpha = (\cos \alpha) \cdot (\tan \alpha)$

$$\sin \alpha = \left(-\frac{\sqrt{2}}{4}\right) \times \sqrt{7} = -\frac{\sqrt{14}}{4}$$

4 - لدينا : $\cos \frac{5\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{2}$

نعلم أن $\cos^2 \frac{5\pi}{12} + \sin^2 \frac{5\pi}{12} = 1$

إذن $\sin^2 \frac{5\pi}{12} + \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)^2 = 1$

$$\sin^2 \frac{5\pi}{12} = 1 - \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)^2$$

$$= 1 - \frac{8 - 2\sqrt{12}}{16}$$

$$= \frac{16 - 8 + 2\sqrt{12}}{16}$$

$$= \frac{8 + 2\sqrt{12}}{16}$$

$$= \left(\frac{\sqrt{6} + \sqrt{2}}{4}\right)^2$$

إذن $\sin \frac{5\pi}{12} = -\frac{\sqrt{6} + \sqrt{2}}{4}$

أو $\sin \frac{5\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4}$

تمرين 10

1 - احسب التعابير التالية :

$$A = \sin^6 x + \cos^6 x - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$B = 2(\cos^6 x + \sin^6 x) - 3(\cos^4 x + \sin^4 x)$$

$$C = \sin^8 x + \cos^8 x + 6\sin^4 x \cdot \cos^4 x + 4\sin^2 x \cos^2 x (\sin^4 x + \cos^4 x)$$

$$D = \sqrt{\sin^4 x + 4\cos^2 x} + \sqrt{\cos^4 x + 4\sin^2 x}$$

$$E = \frac{\cos x}{1 + \sin x} + \frac{\sin x}{1 + \cos x} + \frac{(1 - \sin x)(1 - \cos x)}{\sin x \cdot \cos x}$$

الجواب :

1 - لدينا :

$$A = \sin^6 x + \cos^6 x - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$= (\sin^2 x)^3 + (\cos^2 x)^3 - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$= (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$= \sin^4 x + \cos^4 x - \sin^2 x \cos^2 x - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$= -\sin^4 x + \sin^2 x - \sin^2 x \cdot \cos^2 x$$

$$= \sin^2 x (1 - \sin^2 x) - \sin^2 x \cdot \cos^2 x$$

$$= \sin^2 x \cdot \cos^2 x - \sin^2 x \cdot \cos^2 x$$

$$= 0$$

$$A = 0$$

$$B = 2(\cos^6 x + \sin^6 x) - 3(\cos^4 x + \sin^4 x)$$

$$= 2(\cos^2 x + \sin^2 x)(\cos^4 x + \sin^4 x - \sin^2 x \cos^2 x) - 3(\cos^4 x + \sin^4 x)$$

$$= 2\cos^4 x + 2\sin^4 x - 2\sin^2 x \cdot \cos^2 x - 3\cos^4 x - 3\sin^4 x$$

$$= -\cos^4 x - \sin^4 x - 2\sin^2 x \cdot \cos^2 x$$

$$= - (\cos^4 x + \sin^4 x + 2\sin^2 x \cdot \cos^2 x)$$

$$= - (\cos^2 x + \sin^2 x)^2 = - 1$$

$$B = - 1$$

$$C = \sin^8 x + \cos^8 x + 6\sin^4 x \cdot \cos^4 x + 4\sin^2 x \cos^2 x (\sin^4 x + \cos^4 x)$$

$$= (\sin^4 x + \cos^4 x)^2 - 2\sin^4 x \cos^4 x + 6 \sin^4 x \cos^4 x + 4 \sin^2 x \cos^2 x (\sin^4 x + \cos^4 x)$$

$$= (\sin^4 x + \cos^4 x)^2 + 4 \sin^2 x \cos^2 x (\sin^4 x + \cos^4 x) + 4 \sin^4 x \cdot \cos^4 x$$

$$= (\sin^4 x + \cos^4 x) (\sin^4 x + \cos^4 x + 2\sin^2 x \cdot \cos^2 x) + 2\sin^2 x \cos^2 x (\sin^4 x + \cos^4 x) + 4\sin^4 x \cdot \cos^4 x$$

$$= (\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)^2 + 2\sin^6 x \cos^2 x + 2\sin^2 x \cdot \cos^6 x + 4\sin^4 x \cdot \cos^4 x$$

$$= \sin^4 x + \cos^4 x + (2\sin^6 x \cdot \cos^2 x + 2\sin^4 x \cos^4 x) + (2\sin^2 x \cos^6 x + 2\sin^4 x \cos^4 x)$$

$$= \sin^4 x + \cos^4 x + 2\sin^4 x \cos^2 x + 2\cos^4 x \cdot \sin^2 x (\cos^2 x + \sin^2 x)$$

$$= \sin^4 x + \cos^4 x + 2 \cos^2 x \cdot \sin^2 x (\sin^2 x + \cos^2 x)$$

$$= (\sin^2 x)^2 + (\cos^2 x)^2 + 2\sin^2 x \cdot \cos^2 x$$

$$= (\sin^2 x + \cos^2 x)^2$$

$$= 1^2$$

$$= 1$$

$$C = 1$$

$$D = \sqrt{\sin^4 x + 4\cos^2 x} + \sqrt{\cos^4 x + 4\sin^2 x}$$

$$= \sqrt{\sin^4 x + 4(1 - \sin^2 x)} + \sqrt{\cos^4 x + 4(1 - \cos^2 x)}$$

$$= \sqrt{\sin^4 x - 4\sin^2 x + 4} + \sqrt{\cos^4 x - 4\cos^2 x + 4}$$

$$= \sqrt{(\sin^2 x - 2)^2} + \sqrt{(\cos^2 x - 2)^2}$$

$$= |\sin^2 x - 2| + |\cos^2 x - 2|$$

$$= 2 - \sin^2 x + 2 - \cos^2 x$$

$$- 1 \leq \sin x \leq 1 \quad \text{و} \quad - 1 \leq \cos x \leq 1 \quad \text{لأن}$$

$$0 \leq \sin^2 x \leq 1 \quad \text{و} \quad 0 \leq \cos^2 x \leq 1 \quad \text{ومنه}$$

$$\sin^2 x - 2 \leq 0 \quad \text{و} \quad \cos^2 x - 2 \leq 0 \quad \text{إذن}$$

$$\begin{aligned} D &= 4 - (\sin^2 x + \cos^2 x) \quad \text{إذن} \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

$$D = 3$$

$$\begin{aligned} E &= \frac{\cos x}{1 + \sin x} + \frac{\sin x}{1 + \cos x} + \frac{(1 - \sin x)(1 - \cos x)}{\sin x \cdot \cos x} \\ &= \frac{\cos x (1 - \sin x)}{(1 + \sin x)(1 - \sin x)} + \frac{\sin x (1 - \cos x)}{(1 + \cos x)(1 - \cos x)} + \frac{1 - \cos x - \sin x + \cos x \sin x}{\sin \cdot \cos x} \\ &= \frac{\cos x (1 - \sin x)}{1 - \sin^2 x} + \frac{\sin x (1 - \cos x)}{1 - \cos^2 x} + \frac{1 - \cos x - \sin x + \sin x \cos x}{\sin \cdot \cos x} \\ &= \frac{\cos x (1 - \sin x)}{\cos^2 x} + \frac{\sin x (1 - \cos x)}{\sin^2 x} + \frac{1 - \cos x - \sin x + \sin x \cos x}{\sin \cdot \cos x} \\ &= \frac{1 - \sin x}{\cos x} + \frac{1 - \cos x}{\sin x} + \frac{1 - \cos x - \sin x + \sin x \cdot \cos x}{\sin \cdot \cos x} \\ &= \frac{\sin x (1 - \sin x) + \cos x (1 - \cos x) + 1 - \cos x - \sin x + \sin x \cos x}{\sin \cdot \cos x} \\ &= \frac{\cancel{\sin x} - \sin^2 x + \cancel{\cos x} - \cos^2 x + 1 - \cancel{\cos x} - \cancel{\sin x} + \sin x \cos x}{\sin \cdot \cos x} \\ &= \frac{- (\sin^2 x + \cos^2 x) + 1 + \sin x \cdot \cos x}{\sin \cdot \cos x} \\ &= \frac{\sin x \cos x}{\sin x \cos x} = 1 \end{aligned}$$

تمرين 11

1 - ليكن x من $]-\pi, \pi[\setminus \{0\}$ و $x \neq \frac{\pi}{2}$ و $x \neq -\frac{\pi}{2}$

بين أن : $\frac{1}{\tan^2(x)} - \cos^2(x) = \cos^2(x) \times \frac{1}{\tan^2(x)}$

2 - ليكن x و y من $[-\pi, \pi]$ و يخالفان $\frac{\pi}{2}$ و $-\frac{\pi}{2}$

بين أن : $\sin^2 x - \sin^2 y = \frac{1}{1 + \tan^2 y} - \frac{1}{1 + \tan^2 x}$

الجواب :

$$\begin{aligned} A &= 2\sin x \cos x (1 - 2\sin^2 x) \quad \text{لدينا} \\ &= 2(\tan x \cos x) \cdot \cos x (1 - 2(1 - \cos^2 x)) \\ &= 2\tan x \cdot \cos^2 x (2\cos^2 x - 1) \\ &= 2\tan x \left(\frac{1}{1 + \tan^2 x} \right) \left(\frac{2}{1 + \tan^2 x} - 1 \right) \\ &= 2\tan x \left(\frac{1}{1 + \tan^2 x} \right) \left(\frac{1 - \tan^2 x}{1 + \tan^2 x} \right) \\ &= \frac{2(\tan x)(1 - \tan^2 x)}{(1 + \tan^2 x)^2} \end{aligned}$$

$$A = \frac{2(\tan x)(1 - \tan^2 x)}{(1 + \tan^2 x)^2}$$

إذن

$$\begin{aligned} B &= \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} \quad \text{* لدينا} \\ &= \frac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\sin x - \cos x} \\ &= \sin^2 x + \sin x \cdot \cos x + \cos^2 x \\ &= 1 - \cos^2 x + \sin x \cdot \cos x + \cos^2 x \\ &= 1 - \sin x \cdot \cos x \\ &= 1 - (\tan x)(\cos x) \cdot \cos x \\ &= 1 - \tan x \times \cos^2 x \\ &= 1 - \tan x \times \frac{1}{1 + \tan^2 x} \\ &= 1 - \frac{\tan x}{1 + \tan^2 x} \\ &= \frac{1 + \tan^2 x - \tan x}{1 + \tan^2 x} \end{aligned}$$

$$B = \frac{\tan^2 x - \tan x + 1}{\tan^2 x + 1}$$

إذن

الجواب :

- 1

$$\frac{1}{\tan^2(x)} - \cos^2 x = \frac{1}{\frac{\sin^2(x)}{\cos^2 x}} - \cos^2 x$$

$$= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$$

$$= \cos^2 x \left(\frac{1}{\sin^2 x} - 1 \right)$$

$$= \cos^2 x \left(\frac{1 - \sin^2 x}{\sin^2 x} \right)$$

$$= \cos^2 x \left(\frac{\cos^2 x}{\sin^2 x} \right)$$

إذن

$$(\cos^2 x) \times \frac{1}{\tan^2 x} = \frac{1}{\tan^2(x)} - \cos^2(x)$$

2 - لدينا

$$\begin{aligned} \sin^2 x - \sin^2 y &= 1 - \cos^2 x - (1 - \cos^2 y) \\ &= -\cos^2 x + \cos^2 y \\ &= \cos^2 y - \cos^2 x \end{aligned}$$

$$= \frac{1}{1 + \tan^2 y} - \frac{1}{1 + \tan^2 x}$$

تمرين 12 :

$$1 - \text{ليكن } x \text{ من المجال } [0, \pi] / \left\{ \frac{\pi}{2} \right\}$$

حدد بدلالة $\tan(x)$ مايلي :

$$A = 2\sin x \cos x (1 - 2\sin^2 x)$$

$$B = \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} ; x \neq \frac{\pi}{4}$$

$$C = \cos^4 x - \sin^4 x + \cos^2 x - \sin^2 x$$

$$\begin{aligned}
 &= \cos\left(4\pi + \frac{2\pi}{3}\right) + \sin\left(4\pi - \frac{\pi}{6}\right) \\
 &- 2\sin\left(4\pi + \frac{\pi}{2}\right) \\
 &= \cos\left(\frac{2\pi}{3}\right) + \sin\left(-\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{2}\right) \\
 &= \cos\left(\pi - \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{2}\right) \\
 &= -\cos\frac{\pi}{3} - \sin\frac{\pi}{6} - 2\sin\frac{\pi}{2} \\
 &= -\frac{1}{2} - \frac{1}{2} - 2 \times 1 \\
 &= -1 - 2
 \end{aligned}$$

$$A = -3$$

إذن

$$\begin{aligned}
 B &= \cos\left(\frac{3\pi}{4}\right) \times \sin\left(\frac{4\pi}{3}\right) \cos\left(\frac{7\pi}{6}\right) \times \\
 &\sin\left(\frac{5\pi}{4}\right) \\
 &= \cos\left(\pi - \frac{\pi}{4}\right) \times \sin\left(\pi + \frac{\pi}{3}\right) \times \\
 &\cos\left(\pi - \frac{\pi}{6}\right) \times \sin\left(\pi + \frac{\pi}{4}\right) \\
 &= \left(-\cos\frac{\pi}{4}\right) \times \left(-\sin\frac{\pi}{3}\right) \times \left(-\cos\frac{\pi}{6}\right) \times \left(-\sin\frac{\pi}{4}\right) \\
 &= \left(\cos\frac{\pi}{4}\right) \times \left(\sin\frac{\pi}{3}\right) \times \left(\cos\frac{\pi}{6}\right) \times \left(\sin\frac{\pi}{4}\right) \\
 &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \times \frac{1}{2} \times \frac{\sqrt{2}}{2} \\
 &= \frac{2 \times \sqrt{2}}{2 \times 8} = \frac{\sqrt{3}}{8}
 \end{aligned}$$

$$B = \frac{\sqrt{3}}{8}$$

لدينا :

$$\begin{aligned}
 C &= \tan\left(\frac{2\pi}{3}\right) \times \tan\left(\frac{5\pi}{4}\right) \times \tan\left(\frac{5\pi}{6}\right) \\
 &= \tan\left(\pi - \frac{\pi}{3}\right) \times \tan\left(\pi + \frac{\pi}{4}\right) \times \tan\left(\pi + \frac{\pi}{6}\right) \\
 &= -\tan\left(\frac{\pi}{3}\right) \times \tan\left(\frac{\pi}{4}\right) \times \tan\left(\frac{\pi}{6}\right) \\
 &= \sqrt{3} \times 1 \times \frac{\sqrt{3}}{3} \\
 &= \frac{-3}{3} \\
 &= -1
 \end{aligned}$$

$$C = -1$$

إذن

* لدينا :

$$\begin{aligned}
 C &= \cos^4 x - \sin^4 x + \cos^2 x - \sin^2 x \\
 &= (\cos^2 x)^2 - (\sin^2 x)^2 + \cos^2 x - \sin^2 x \\
 &= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) + \cos^2 x - \sin^2 x \\
 &= \cos^2 x - \sin^2 x + \cos^2 x - \sin^2 x \\
 &= 2(\cos^2 x - \sin^2 x) \\
 &= 2(\cos^2 x - 1 + \cos^2 x) \\
 &= 2(2\cos^2 x - 1) \\
 &= 2\left(\frac{2}{1 + \tan^2 x} - 1\right) \\
 &= 2\left(\frac{2 - 1 - \tan^2 x}{1 + \tan^2 x}\right)
 \end{aligned}$$

$$C = \frac{2(1 - \tan^2 x)}{1 + \tan^2 x}$$

إذن

تمرين 13 :

احسب مايلي :

$$\begin{aligned}
 A &= \cos\left(\frac{14\pi}{3}\right) + \sin\left(\frac{23\pi}{6}\right) - 2\sin\left(\frac{9\pi}{2}\right) \\
 B &= \cos\left(\frac{3\pi}{4}\right) \times \sin\left(\frac{4\pi}{3}\right) \cos\left(\frac{7\pi}{6}\right) \times \sin\left(\frac{5\pi}{4}\right) \\
 C &= \tan\left(\frac{2\pi}{3}\right) \times \tan\left(\frac{5\pi}{4}\right) \times \tan\left(\frac{5\pi}{6}\right)
 \end{aligned}$$

الجواب :

لدينا :

$$\begin{aligned}
 A &= \cos\left(\frac{12\pi + 2\pi}{3}\right) + \sin\left(\frac{24\pi - \pi}{6}\right) \\
 &- 2\sin\left(\frac{8\pi + \pi}{2}\right)
 \end{aligned}$$

تمرين 14 :

احسب مايلي :

$$A = \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{5\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$$

$$B = \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{2\pi}{7}\right) + \tan\left(\frac{5\pi}{7}\right) + \tan\left(\frac{6\pi}{7}\right)$$

$$C = \cos^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{5\pi}{12}\right) + \cos^2\left(\frac{7\pi}{12}\right) + \cos^2\left(\frac{11\pi}{12}\right)$$

الجواب :

لدينا :

$$A = \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{5\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$$

$$= \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{7\pi - 2\pi}{7}\right) + \cos\left(\frac{7\pi - 2\pi}{7}\right)$$

$$= \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + \cos\left(\pi - \frac{2\pi}{7}\right) + \cos\left(\pi - \frac{\pi}{7}\right)$$

$$= \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) - \cos\left(\frac{2\pi}{7}\right) - \cos\left(\frac{\pi}{7}\right)$$

$$= 0$$

$$A = 0$$

إذن

$$B = \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{2\pi}{7}\right) + \tan\left(\frac{5\pi}{7}\right) + \tan\left(\frac{6\pi}{7}\right)$$

$$= \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{2\pi}{7}\right) + \tan\left(\pi - \frac{2\pi}{7}\right) + \tan\left(\pi - \frac{\pi}{7}\right)$$

$$= \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{2\pi}{7}\right) - \tan\left(\frac{2\pi}{7}\right) - \tan\left(\frac{\pi}{7}\right)$$

$$= 0$$

إذن



$$= 2 \times 1 = 2$$

$$C = 2 \quad \text{إذن}$$

تمرين 15:

نعتبر التعبيرات التالية :

$$A = \cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{3\pi}{5} + \cos\frac{4\pi}{5}$$

$$B = \sin\left(\frac{11\pi}{26}\right) + \sin\left(\frac{3\pi}{26}\right) + \cos\left(\frac{12\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right)$$

$$C = \sin\left(\frac{\pi}{14}\right) + \sin\left(\frac{3\pi}{14}\right) + \cos\left(\frac{5\pi}{14}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{5\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$$

احسب A و B و C.

الجواب:

لدينا :

$$A = \cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{3\pi}{5} + \cos\frac{4\pi}{5}$$

$$= \cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\left(\pi - \frac{2\pi}{5}\right) + \cos\left(\pi - \frac{\pi}{5}\right)$$

$$= \cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{2\pi}{5}\right) - \cos\left(\frac{2\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right)$$

$$= 0 \quad \boxed{A = 0} \quad \text{إذن}$$

$$B = \sin\left(\frac{11\pi}{26}\right) + \sin\left(\frac{3\pi}{26}\right) + \cos\left(\frac{12\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right)$$

لدينا :

$$= \sin\left(\frac{13\pi - 2\pi}{26}\right) + \sin\left(\frac{13\pi - 10\pi}{26}\right) + \cos\left(\pi - \frac{\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right)$$

$$= \sin\left(\frac{\pi}{2} - \frac{\pi}{13}\right) + \sin\left(\frac{\pi}{2} - \frac{5\pi}{13}\right) + \cos\left(\pi - \frac{\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right)$$

$$= \cos\frac{\pi}{13} + \cos\frac{5\pi}{13} - \cos\frac{\pi}{13} - \cos\frac{5\pi}{13}$$

$$= 0 \quad \boxed{B = 0} \quad \text{إذن}$$

لدينا :

$$C = \sin\left(\frac{\pi}{14}\right) + \sin\left(\frac{3\pi}{14}\right) + \sin\left(\frac{5\pi}{14}\right) + \cos\left(\frac{8\pi}{14}\right) + \cos\left(\frac{10\pi}{14}\right) + \cos\left(\frac{12\pi}{14}\right)$$

$$= \sin\left(\frac{\pi}{14}\right) + \sin\left(\frac{3\pi}{14}\right) + \sin\left(\frac{5\pi}{14}\right) + \cos\left(\frac{7\pi + \pi}{14}\right) + \cos\left(\frac{7\pi + 3\pi}{14}\right) + \cos\left(\frac{7\pi + 5\pi}{14}\right)$$

$$= \sin\left(\frac{\pi}{14}\right) + \sin\left(\frac{3\pi}{14}\right) + \sin\left(\frac{5\pi}{14}\right) + \cos\left(\frac{\pi}{2} + \frac{\pi}{14}\right) + \cos\left(\frac{\pi}{2} + \frac{3\pi}{14}\right) + \cos\left(\frac{\pi}{2} + \frac{5\pi}{14}\right)$$

$$= \cancel{\sin\left(\frac{\pi}{14}\right)} + \cancel{\sin\left(\frac{3\pi}{14}\right)} + \cancel{\sin\left(\frac{5\pi}{14}\right)} - \cancel{\sin\left(\frac{\pi}{14}\right)} - \cancel{\sin\frac{3\pi}{14}} - \cancel{\sin\frac{5\pi}{14}}$$

$$= 0 \quad \boxed{C = 0} \quad \text{إذن}$$

تمرين 16:

احسب مايلي :

$$A = \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8}$$

$$B = \sin^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{9} + \cos^2 \frac{6\pi}{7} + \cos^2 \frac{8\pi}{9}$$

الجواب :

$$A = \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8}$$

$$= \sin^2 \left(\frac{\pi}{8} \right) + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) + \sin^2 \left(\frac{\pi}{2} + \frac{\pi}{8} \right) + \sin^2 \left(\pi - \frac{\pi}{8} \right)$$

$$= \sin^2 \left(\frac{\pi}{8} \right) + \cos^2 \left(\frac{\pi}{8} \right) + \left(-\cos \frac{\pi}{8} \right)^2 + \sin^2 \left(\frac{\pi}{8} \right)$$

$$= \sin^2 \left(\frac{\pi}{8} \right) + \cos^2 \left(\frac{\pi}{8} \right) + \cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}$$

$$= 2 \left(\sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} \right)$$

$$= 2 \times 1 = 2$$

$$\boxed{A = 2} \quad \text{إذن}$$

$$B = \sin^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{9} + \cos^2 \frac{6\pi}{7} + \cos^2 \frac{8\pi}{9}$$

$$= \sin^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{9} + \cos^2 \left(\pi - \frac{\pi}{7} \right) + \cos^2 \left(\pi - \frac{\pi}{9} \right)$$

$$= \sin^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{9} + \left(-\cos \frac{\pi}{7} \right)^2 + \left(-\cos \frac{\pi}{9} \right)^2$$

$$= \sin^2 \left(\frac{\pi}{7} \right) + \sin^2 \left(\frac{\pi}{9} \right) + \cos^2 \left(\frac{\pi}{7} \right) + \cos^2 \left(\frac{\pi}{9} \right)$$

$$= \left(\sin^2 \left(\frac{\pi}{7} \right) + \cos^2 \left(\frac{\pi}{7} \right) \right) + \left(\sin^2 \left(\frac{\pi}{9} \right) + \cos^2 \left(\frac{\pi}{9} \right) \right)$$

$$= 1 + 1 = 2$$

$$\boxed{B = 2}$$

تمرين 17

(2) - لدينا $\cos\left(-\frac{\pi}{8}\right) = \cos\left(\frac{\pi}{8}\right)$

$$= \frac{\sqrt{2+\sqrt{2}}}{2}$$

إذن $\cos\left(-\frac{\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$

$$\sin\left(-\frac{7\pi}{8}\right) = -\sin\left(\frac{7\pi}{8}\right) = -\sin\left(\pi - \frac{\pi}{8}\right)$$

$$= -\sin\left(\frac{\pi}{8}\right)$$

$$= -\frac{\sqrt{2-\sqrt{2}}}{2}$$

إذن $\sin\left(-\frac{7\pi}{8}\right) = -\frac{\sqrt{2-\sqrt{2}}}{2}$

(3) - $\cos\left(\frac{513\pi}{8}\right)$

$$= \cos\left(\frac{512\pi + \pi}{8}\right)$$

$$= \cos\left(64\pi + \frac{\pi}{8}\right)$$

$$= \cos\left(\frac{\pi}{8}\right)$$

$$= \frac{\sqrt{2+\sqrt{2}}}{2}$$

إذن $\cos\left(\frac{513\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$

لدينا $\sin\left(\frac{37\pi}{8}\right)$

$$= \sin\left(\frac{40\pi - 3\pi}{8}\right)$$

$$= \sin\left(5\pi - \frac{3\pi}{8}\right)$$

$$= \sin\left(\pi - \frac{3\pi}{8}\right)$$

نعلم أن : $\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$

(1) - احسب $\cos\frac{3\pi}{8}$, $\sin\frac{\pi}{8}$

(2) - استنتج : $\sin\left(-\frac{7\pi}{8}\right)$, $\sin\left(-\frac{\pi}{8}\right)$

(3) - احسب $\sin\left(\frac{37\pi}{8}\right)$ و $\cos\left(\frac{513\pi}{8}\right)$

و $\tan\left(\frac{25\pi}{8}\right)$

الجواب :

(1) - نعلم أن : $\cos^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{\pi}{8}\right) = 1$

أي أن $\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)^2 + \sin^2\left(\frac{\pi}{8}\right) = 1$

$$\sin^2\left(\frac{\pi}{8}\right) = 1 - \frac{2+\sqrt{2}}{4}$$

$$= \frac{4-2-\sqrt{2}}{4}$$

$$= \frac{2-\sqrt{2}}{4}$$

$\sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$ أو $\sin\frac{\pi}{8} = -\frac{\sqrt{2-\sqrt{2}}}{2}$

بما أن $0 < \frac{\pi}{8} < \frac{\pi}{2}$ فإن $\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$

$$\cos\left(\frac{3\pi}{8}\right) = \cos\left(\frac{4\pi - \pi}{8}\right)$$

$$= \cos\left(\frac{\pi}{2} - \frac{\pi}{8}\right)$$

$$= \sin\frac{\pi}{8}$$

$$= \frac{\sqrt{2-\sqrt{2}}}{2}$$

إذن $\cos\left(\frac{3\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$

تمرين 17

ليكن x من المجال $[0, \pi]$ و $x \neq \frac{\pi}{2}$

نضع

$$P(x) = 2\left[1 - \cos^2\left(\frac{\pi}{2} + x\right)\right] - \cos(\pi - x) \sin(\pi + x)$$

1 - بسط $P(x)$ ثم احسب $P(0)$ و $P\left(\frac{\pi}{4}\right)$.

$$2 - \text{أ - بين أن : } P(x) = \frac{2 - \tan x}{1 + \tan^2 x}$$

ب - احسب $P\left(\frac{\pi}{3}\right)$ و $P(x)$.

3 - إذا علمت أن $P(x) = 2$ و $x \in \left[0, \frac{\pi}{2}\right]$

فاحسب $\tan x$ واستنتج x .

الجواب :

- 1

$$P(x) = 2\left[1 - \cos^2\left(\frac{\pi}{2} + x\right)\right] - \cos(\pi - x) \sin(\pi + x)$$

$$= 2\left[1 - (-\sin x)^2\right] - (-\cos x) (-\sin x)$$

$$= 2(1 - \sin^2 x) - (-\cos x) \times (-\sin x)$$

$$= 2\cos^2 x - \cos x \cdot \sin x$$

$$P(x) = 2\cos^2 x - \cos x \cdot \sin x \quad \text{إذن}$$

$$P(0) = 2\cos^2 0 - (\cos 0) \times (\sin 0) \quad \text{لدينا}$$

$$= 2 \times 1 - 1 \times 0$$

$$= 2$$

$$P(0) = 2 \quad \text{إذن}$$

$$P\left(\frac{\pi}{4}\right) = 2\cos^2\frac{\pi}{4} - \cos\left(\frac{\pi}{4}\right) \times \sin\left(\frac{\pi}{4}\right)$$

$$= \sin\left(\frac{3\pi}{8}\right)$$

$$= \sin\left(\frac{3\pi}{8}\right)$$

$$= \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right)$$

$$= \cos\frac{\pi}{8}$$

$$\sin\left(\frac{37\pi}{8}\right) = \frac{\sqrt{2} + \sqrt{2}}{2} \quad \text{إذن}$$

لدينا

$$\tan\left(\frac{25\pi}{8}\right) = \tan\left(\frac{24\pi + \pi}{8}\right)$$

$$= \tan\left(3\pi + \frac{\pi}{8}\right)$$

$$= \tan\frac{\pi}{8}$$

$$= \frac{\sqrt{2} - \sqrt{2}}{2}$$

$$= \frac{\sqrt{2} + \sqrt{2}}{2}$$

$$= \frac{\sqrt{2} - \sqrt{2}}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{\sqrt{2} - \sqrt{2} \cdot \sqrt{2} - \sqrt{2}}{\sqrt{4} - 2}$$

$$= \frac{2 - \sqrt{2}}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} - 1$$

$$= \sqrt{2} - 1$$

$$= \sqrt{2} - 1$$

$$\tan\frac{\pi}{8} = \sqrt{2} - 1 \quad \text{إذن}$$

$$\frac{2 - \tan x}{1 + \tan^2 x} = 2 \quad \text{لدينا } P(x) = 2 \text{ تكافئ}$$

$$2 - \tan x = 2 + 2\tan^2 x \quad \text{تكافئ}$$

$$-\tan x = 2\tan^2 x$$

$$2\tan^2 x + \tan x = 0 \quad \text{أي أن}$$

$$\tan x (2\tan x + 1) = 0$$

$$\tan x = 0 \text{ أو } \tan x = -\frac{1}{2}$$

$$\text{وبما أن } x \in [0, \frac{\pi}{2}] \text{ فإن } \tan x \geq 0$$

$$\text{إذن } \tan x = 0$$

$$\text{وبما أن } x \in [0, \frac{\pi}{2}]$$

$$\text{فإن } x = 0$$

تمرين 18:

ليكن x من المجال $]-\pi; \pi[$ و $x \neq \frac{\pi}{2}$

$$\text{و } x \neq -\frac{\pi}{2}$$

$$P(x) = \frac{1 - \sin x}{1 + \sin x} + \frac{1 + \sin x}{1 - \sin x} \quad \text{و}$$

$$P(x) = 2(1 + 2\tan^2 x) \quad \text{بين أن } -1$$

$$P\left(\frac{\pi}{6}\right) \text{ و } P\left(\frac{\pi}{3}\right) \text{ احسب } -1 - 2$$

$$P\left(-\frac{\pi}{3}\right) \text{ و } P\left(\frac{5\pi}{6}\right) \text{ و } P\left(\frac{2\pi}{3}\right) \text{ استنتج } -1$$

$$\text{و } P\left(-\frac{\pi}{6}\right)$$

$$-3 \text{ إذا علمت أن } P(x) = 14$$

$$\text{و } 0 < x < \frac{\pi}{2} \text{ فاحسب } \tan x \text{ و } \cos x \text{ ثم } \sin x$$

$$= 2\left(\frac{\sqrt{2}}{2}\right)^2 - \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2}$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2} \quad P\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

- 1 - 2

$$P(x) = 2\cos^2 x - \cos x \cdot \sin x \quad \text{لدينا}$$

$$= 2\cos^2 x - \tan x \cdot \cos x \cdot \cos x$$

$$= 2\cos^2 x - (\tan x) \cdot \cos^2 x$$

$$= \cos^2 x \cdot (2 - \tan x)$$

$$= \frac{1}{1 + \tan^2 x} (2 - \tan x)$$

$$P(x) = \frac{2 - \tan x}{1 + \tan^2 x}$$

إذن

$$P\left(\frac{\pi}{3}\right) = \frac{2 - \tan \frac{\pi}{3}}{2 + \tan^2 \frac{\pi}{3}}$$

ب - لدينا

$$= \frac{2 - \sqrt{3}}{1 + (\sqrt{3})^2}$$

$$P\left(\frac{\pi}{3}\right) = \frac{2 - \sqrt{3}}{4}$$

$$P(\pi) = \frac{2 - \tan \pi}{1 + \tan^2 \pi}$$

$$= \frac{2 - 0}{1 + 0^2}$$

$$= 2$$

$$P(\pi) = 2$$

إذن

$$P(x) = 2; x \in \left[0, \frac{\pi}{2}\right[- 3$$

$$= 2\left(1 + \frac{2}{3}\right)$$

$$= \frac{10}{3}$$

$$P\left(\frac{\pi}{6}\right) = \frac{10}{3}$$

ذن

$$P(x) = P(\pi - x) = P(-x) \quad \text{ب - لدينا}$$

$$P\left(\frac{2\pi}{3}\right) = P\left(\pi - \frac{\pi}{3}\right)$$

$$= P\left(\frac{\pi}{3}\right)$$

لدينا

$$P\left(\frac{2\pi}{3}\right) = 14$$

اذن

$$P\left(\frac{5\pi}{6}\right) = P\left(\pi - \frac{\pi}{6}\right)$$

$$= P\left(\frac{\pi}{6}\right)$$

لدينا

$$P\left(\frac{5\pi}{6}\right) = \frac{10}{3}$$

اذن

$$P\left(-\frac{\pi}{3}\right) = P\left(\frac{\pi}{3}\right)$$

$$= 14$$

لدينا

$$P\left(-\frac{\pi}{6}\right) = P\left(\frac{\pi}{6}\right) = \frac{10}{3}$$

$$P(x) = 14$$

3 - لدينا :

$$2(1 + 2\tan^2 x) = 14$$

$$1 + 2\tan^2 x = 7$$

$$2\tan^2 x = 6$$

اذن

$$\tan^2 x = 3$$

تكافي

$$\tan x = \sqrt{3} \text{ أو } \tan x = -\sqrt{3}$$

$$\tan x > 0 \text{ فإذن } 0 < x < \frac{\pi}{2}$$

الجواب :

$$P(x) = \frac{1 - \sin x}{1 + \sin x} + \frac{1 + \sin x}{1 - \sin x} \quad \text{لدينا : - 1}$$

$$= \frac{(1 - \sin x)^2 + (1 + \sin x)^2}{(1 + \sin x)(1 - \sin x)}$$

$$= \frac{1 + \sin^2 x - 2\sin x + 1 + \sin^2 x + 2\sin x}{1 - \sin^2 x}$$

$$= \frac{2 + 2\sin^2 x}{\cos^2 x}$$

$$= \frac{2 + 2(1 - \cos^2 x)}{\cos^2 x}$$

$$= \frac{4 - 2\cos^2 x}{\cos^2 x}$$

$$= \frac{2\left(2 - \frac{1}{1 + \tan^2 x}\right)}{\frac{1}{1 + \tan^2 x}}$$

$$= \frac{2\left(\frac{2 + 2\tan^2 x - 1}{1 + \tan^2 x}\right)}{\frac{1}{1 + \tan^2 x}}$$

$$P(x) = 2(1 + 2\tan^2 x) \quad \text{اذن}$$

$$P\left(\frac{\pi}{3}\right) = 2\left(1 + 2\tan^2 \frac{\pi}{3}\right) \quad \text{- 1 - 2}$$

$$= 2(1 + 2(\sqrt{3})^2)$$

$$= 2(1 + 6)$$

$$P\left(\frac{\pi}{3}\right) = 14 \quad \text{اذن}$$

$$P\left(\frac{\pi}{6}\right) = 2\left(1 + 2\tan^2 \frac{\pi}{6}\right)$$

$$= 2\left(1 + 2 \times \left(\frac{\sqrt{3}}{3}\right)^2\right)$$

الجواب :

لدينا

$$A(x) = \cos^2 x + 3 \sin x \cdot \cos x - 2 \sin^2 x$$

$$= \cos^2 x \left(1 + \frac{3 \sin x \cdot \cos x}{\cos^2 x} - \frac{2 \sin^2 x}{\cos^2 x} \right)$$

$$= \cos^2 x \left(1 + \frac{3 \sin x}{\cos x} - 2 \cdot \left(\frac{\sin x}{\cos x} \right)^2 \right)$$

$$A(x) = \cos^2 x (1 + 3 \tan x - 2 \tan^2 x) \quad \text{إذن}$$

$$\cos x = \frac{\sqrt{5}}{5} \quad \text{لدينا} - 2$$

$$\cos^2 x = \frac{1}{5} \quad \text{إذن}$$

$$\frac{1}{\cos^2 x} = 5 \quad \text{ومنه}$$

$$1 + \tan^2 x = 5 \quad \text{أي أن}$$

$$\tan^2 x = 4 \quad \text{إذن}$$

$$\tan x = -2 \quad \text{أو} \quad \tan x = 2 \quad \text{ومنه}$$

$$0 < x < \frac{\pi}{2} \quad \text{وبما أن}$$

$$\tan x > 0 \quad \text{فإن}$$

$$\tan x = 2 \quad \text{إذن}$$

ومنه

$$A(x) = \frac{1}{5} (1 + 3 \times 2 - 2 \times 4)$$

$$= \frac{1}{5} (1 + 6 - 8)$$

$$= \frac{1}{5}$$

$$A(x) = \frac{1}{5} \quad \text{إذن}$$

$$\tan x = \sqrt{3} \quad \text{ومنه}$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x} \quad \text{لدينا}$$

$$= \frac{1}{1 + (\sqrt{3})^2}$$

$$= \frac{1}{4}$$

$$\cos x = \sqrt{\frac{1}{4}} \quad \text{أو} \quad \cos x = -\sqrt{\frac{1}{4}}$$

$$\cos x = \frac{1}{2} \quad \text{أو} \quad \cos x = -\frac{1}{2} \quad \text{إذن}$$

$$\cos x = \frac{1}{2} \quad \text{فإن} \quad \cos x > 0$$

$$\tan x = \frac{\sin x}{\cos x} \quad \text{لدينا}$$

$$\sin x = \tan x \cos x$$

$$= \sqrt{3} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{\sqrt{3}}{2} \quad \text{إذن}$$

تمرين 19 :

ليكن x عددا حقيقيا بحيث : $x \neq \frac{\pi}{2} + k\pi$

$$k \in \mathbb{Z} ;$$

$$A(x) = \cos^2 x + 3 \sin x \cdot \cos x - 2 \sin^2 x$$

- 1 بين أن

$$A(x) = \cos^2 x (1 + 3 \tan x - 2 \tan^2 x)$$

- 2 احسب $A(x)$ إذا علمت أن :

$$0 < x < \frac{\pi}{2} \quad \text{و} \quad \cos x = \frac{\sqrt{5}}{5}$$

$$A\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) - \cos\left(-\frac{\pi}{2}\right) \quad \text{لدينا}$$

$$= -\sin\frac{\pi}{2} - \cos\frac{\pi}{2}$$

$$= -1 - 0$$

$$\boxed{A\left(-\frac{\pi}{2}\right) = -1}$$

إذن

$$A\left(\frac{13\pi}{3}\right) = \sin\left(\frac{13\pi}{3}\right) - \cos\left(\frac{13\pi}{3}\right) \quad \text{لدينا}$$

$$= \sin\left(4\pi + \frac{\pi}{3}\right) - \cos\left(4\pi + \frac{\pi}{3}\right)$$

$$= \sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$= \frac{\sqrt{3} - 1}{2}$$

$$\boxed{A\left(\frac{13\pi}{3}\right) = \frac{\sqrt{3} - 1}{2}}$$

إذن

$$A(x) = \sin x - \cos x \quad \text{لدينا} \quad 3 -$$

$$[A(x)]^2 = (\sin x - \cos x)^2$$

$$= \sin^2 x - 2\sin x \cdot \cos x + \cos^2 x$$

$$= 1 - 2\sin x \cdot \cos x$$

$$= 1 - 2\tan x \cdot \cos x \cdot \cos x$$

$$= 1 - 2 \cdot \tan x \cdot \cos^2 x$$

$$= 1 - 2\tan x \times \frac{1}{1 + \tan^2 x}$$

$$= 1 - \frac{2\tan x}{1 + \tan^2 x}$$

$$= \frac{1 + \tan^2 x - 2\tan x}{1 + \tan^2 x}$$

$$[A(x)]^2 = \frac{(1 - \tan x)^2}{1 + \tan^2 x}$$

إذن

تمرين 20:

ليكن x من \mathbb{R} . نضع

$$A(x) = 3\cos(3\pi + x) - 2\sin(\pi + x)$$

$$- \cos\left(\frac{9\pi}{2} + x\right) + 2\sin\left(\frac{9\pi}{2} - x\right)$$

1 - احسب $A(x)$ بدلالة $\sin x$ و $\cos x$.

2 - احسب $A(0)$ و $A\left(-\frac{\pi}{2}\right)$ و $A\left(\frac{13\pi}{3}\right)$.

3 - احسب $[A(x)]^2$ بدلالة $\tan x$ لكل x

بخلاف $\frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$.

الجواب:

1 - لدينا

$$A(x) = 3\cos(3\pi + x) - 2\sin(\pi + x) +$$

$$\cos\left(\frac{9\pi}{2} + x\right) + 2\sin\left(\frac{9\pi}{2} - x\right)$$

$$= 3\cos(2\pi + \pi + x) + 2\sin x +$$

$$\cos\left(\frac{4\pi + \pi}{2} + x\right) + 2\sin\left(\frac{8\pi + \pi}{2} - x\right)$$

$$= -3\cos x + 2\sin x + \cos\left(\frac{\pi}{2} + x\right) +$$

$$2\sin\left(\frac{\pi}{2} - x\right)$$

$$= -3\cos x + 2\sin x - \sin x + 2\cos x$$

$$= \sin x - \cos x$$

$$\boxed{A(x) = \sin x - \cos x} \quad \text{إذن}$$

$$A(0) = \sin 0 - \cos 0 \quad \text{لدينا} \quad 2 -$$

$$A(0) = -1 \quad \text{إذن}$$

$$\begin{aligned}
 &= -2 + 3\cos^2x + 3\cos^2x \times \tan x \\
 &= -2 + 3\cos^2x(1 + \tan x) \\
 &= -2 + 3 \times \frac{1}{1 + \tan^2x} (1 + \tan x)
 \end{aligned}$$

$$E(x) = -2 + 3 \times \left(\frac{1 + \tan x}{1 + \tan^2 x} \right)$$

ب - لدينا $E(x) = 1$

تكافئ $-2 + 3 \frac{1 + \tan x}{1 + \tan^2 x} = 1$

أي أن $3 \left(\frac{1 + \tan x}{1 + \tan^2 x} \right) = 3$

ومنه $\frac{1 + \tan x}{1 + \tan^2 x} = 1$

إذن $1 + \tan x = 1 + \tan^2 x$

أي أن $\tan x = \tan^2 x$

أي أن $\tan x - \tan^2 x = 0$

أي أن $\tan x (1 - \tan x) = 0$

إذن $\tan x = 0$ أو $\tan x = 1$

وبما أن $x \neq 0$ فإن $\tan x \neq 0$ و $x \in]0, \frac{\pi}{2}[$

إذن $\tan x = 1$

لدينا $\cos^2 x = \frac{1}{1 + \tan^2 x}$

$$= \frac{1}{1 + 1}$$

$$\cos^2 x = \frac{1}{2}$$

$\cos x = \frac{\sqrt{2}}{2}$ أو $\cos x = -\frac{\sqrt{2}}{2}$

وبما أن $0 < x < \frac{\pi}{2}$ فإن $\cos x = \frac{\sqrt{2}}{2}$

تمرين 21:

ليكن x من $]0; \pi]$. نضع

$$E(x) = \cos^2 x + 3\cos x \cdot \sin x - 2\sin^2 x$$

1 - احسب $E(0)$ و $E(\pi)$.

2 - ليكن x من المجال $]0, \frac{\pi}{2}[$.

أ - بين أن $E(x) = -2 + 3 \left(\frac{1 + \tan x}{1 + \tan^2 x} \right)$

ب - إذا علمت أن $E(x) = 1$.

فاحسب $\tan x$ ثم $\cos x$.

الجواب:

1 - لدينا

$$E(x) = \cos^2 x + 3\cos x \cdot \sin x - 2\sin^2 x$$

$$E(0) = \cos^2 0 + 3\cos 0 \cdot \sin 0 - 2\sin^2 0$$

$$= 1 + 3 \times 0 \times 1 - 2 \times 0^2$$

$$E(0) = 1$$

$$E(\pi) = \cos^2 \pi + 3(\cos \pi) \times (\sin \pi) - 2\sin^2 \pi$$

$$= (-1)^2 + 3(-1) \cdot 0 - 2 \cdot 0^2$$

$$= 1$$

$$E(\pi) = 1$$

إذن

2 - أ - لدينا

$$E(x) = \cos^2 x + 3\cos x \cdot \sin x - 2\sin^2 x$$

$$= \cos^2 x + 3\cos x \times \tan x \cdot \cos x - 2(1 - \cos^2 x)$$

$$= \cos^2 x + 3\cos^2 x \times \tan x - 2 + 2\cos^2 x$$

تمرين 22:

ليكن x من \mathbb{R} . نضع

$$A(x) = \cos^3 x + \sin^3 x + \cos(7\pi + x) - \sin(x - 9\pi)$$

1 - بين أن :

$$A(x) = -(\sin(x) + \cos(x)) \cdot \sin(x) \cdot \cos(x)$$

2 - احسب :

$$A\left(\frac{\pi}{3}\right) \text{ و } A\left(\frac{\pi}{4}\right) \cdot A(0)$$

3 - أ - بين أن :

$$A\left(\frac{\pi}{2} - x\right) = A(x)$$

ب - استنتج حساب : $A\left(\frac{\pi}{6}\right)$ و $A\left(\frac{\pi}{2}\right)$

الجواب :

1 - لدينا :

$$A(x) = \cos^3 x + \sin^3 x + \cos(6\pi + \pi + x) + \sin(x - \pi - 8\pi)$$

$$= \cos^3 x + \sin^3 x + \cos(\pi + x) + \sin(x - \pi)$$

$$= (\cos x + \sin x) \times (\cos^2 x - \sin x \cos x +$$

$$\sin^2 x) - \cos x - \sin x$$

$$= (\cos x + \sin x) \times (1 - \sin x \cos x) -$$

$$(\cos x + \sin x)$$

$$= (\cos x + \sin x) \times (1 - \sin x \cdot \cos x - 1)$$

$$= (\cos x + \sin x) (-\sin x \cdot \cos x)$$

إذن

$$A(x) = -(\cos x + \sin x) \times \sin x \cdot \cos x$$

2 - لدينا

$$A(0) = -(\sin 0 + \cos 0) \times \sin 0 \times \cos 0$$

$$= -1 \times 0 \times 1$$

$$= 0$$

$$A(0) = 0$$

إذن

لدينا

$$A\left(\frac{\pi}{4}\right) = -\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) \times \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{4}$$

$$= -\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2}$$

$$= -\sqrt{2} \times \frac{2}{4}$$

$$= -\frac{\sqrt{2}}{2}$$

$$A\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

إذن

لدينا

$$A\left(\frac{\pi}{3}\right) = -\left(\sin\frac{\pi}{3} + \cos\frac{\pi}{3}\right) \times \sin\frac{\pi}{3} \cdot \cos\frac{\pi}{3}$$

$$= -\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= -\left(\frac{\sqrt{3} + 1}{2}\right) \times \frac{\sqrt{3}}{4}$$

$$A\left(\frac{\pi}{3}\right) = \frac{-(3 + \sqrt{3})}{8}$$

إذن

3 - أ - لدينا :

$$A\left(\frac{\pi}{2} - x\right) = -\left[\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)\right] \cdot$$

$$\sin\left(\frac{\pi}{2} - x\right) \times \cos\left(\frac{\pi}{2} - x\right)$$

$$= -(\cos x + \sin x) \cos x \sin x$$

$$= A(x)$$

$$\sin^2\left(\frac{\pi}{5}\right) = 1 - \frac{6 + 2\sqrt{5}}{16}$$

أي أ-

$$\sin^2\frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{16}$$

$$\sin\frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\sin\frac{\pi}{5} = -\frac{\sqrt{10-2\sqrt{5}}}{4}$$

أو

$$0 < \frac{\pi}{5} < \frac{\pi}{2}$$

وبما أن

$$\sin\frac{\pi}{5} > 0$$

فإن

$$\boxed{\sin\left(\frac{\pi}{5}\right) = \frac{\sqrt{10-2\sqrt{5}}}{4}}$$

ومنه

$$\cos\frac{4\pi}{5} = \cos\left(\frac{5\pi-\pi}{5}\right)$$

لدينا

$$= \cos\left(\pi - \frac{\pi}{5}\right)$$

$$= -\cos\frac{\pi}{5}$$

$$= -\frac{\sqrt{5+1}}{4}$$

$$\sin\frac{4\pi}{5} = \sin\left(\frac{5\pi-\pi}{5}\right)$$

لدينا

$$= \sin\left(\pi - \frac{\pi}{5}\right)$$

$$= +\sin\frac{\pi}{5}$$

$$= +\frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\boxed{\sin\frac{4\pi}{5} = +\frac{\sqrt{10-2\sqrt{5}}}{4}}$$

$$\cos\left(\frac{7\pi}{10}\right) = \cos\left(\frac{5\pi}{10} + \frac{2\pi}{10}\right)$$

- 2

$$\boxed{A\left(\frac{\pi}{2} - x\right) = A(x)}$$

إذن

$$\frac{\pi}{2} - 0 - \frac{\pi}{2} \quad \text{ب - لدينا}$$

$$A\left(\frac{\pi}{2} - 0\right) = A\left(\frac{\pi}{2}\right) \quad \text{إذن}$$

$$A(0) = A\left(\frac{\pi}{2}\right) \quad \text{ومنه}$$

$$A\left(\frac{\pi}{2}\right) = 0 \quad \text{إذن}$$

$$\frac{\pi}{6} - \frac{\pi}{2} - \frac{\pi}{3} \quad \text{لدينا}$$

$$A\left(\frac{\pi}{6}\right) = A\left(\frac{\pi}{2} - \frac{\pi}{3}\right) \quad \text{إذن}$$

$$A\left(\frac{\pi}{6}\right) = A\left(\frac{\pi}{3}\right) \quad \text{ومنه}$$

$$A\left(\frac{\pi}{3}\right) = \frac{-(3 + \sqrt{3})}{8} \quad \text{وبما أن}$$

$$A\left(\frac{\pi}{6}\right) = \frac{-(3 + \sqrt{3})}{8} \quad \text{فإن}$$

تمرين 23

$$\text{علما أن : } \cos\left(\frac{\pi}{5}\right) = \frac{\sqrt{5+1}}{4}$$

$$1 - \text{أحسب } \sin\left(\frac{4\pi}{5}\right), \cos\frac{4\pi}{5}, \sin\frac{\pi}{5}$$

$$2 - \text{أحسب } \tan\frac{3\pi}{10}, \sin\frac{-3\pi}{10}, \cos\frac{7\pi}{10}$$

$$3 - \cos\frac{101\pi}{10}; \sin\frac{-84\pi}{10}$$

الجواب :

1 - نعلم أن

$$\cos^2\left(\frac{\pi}{5}\right) + \sin^2\left(\frac{\pi}{5}\right) = 1$$

$$\left(\frac{\sqrt{5+1}}{4}\right)^2 + \sin^2\left(\frac{\pi}{5}\right) = 1$$

تكافئ

$$= \frac{(\sqrt{5} + 5)\sqrt{2} \cdot \sqrt{5 + \sqrt{5}}}{20}$$

$$= \frac{\sqrt{2} \times (\sqrt{5 + \sqrt{5}}^3)}{20}$$

3 - لدينا

$$\sin\left(-\frac{84\pi}{5}\right) = \sin\left(-\frac{85\pi + \pi}{5}\right)$$

$$= \sin\left(-17\pi + \frac{\pi}{5}\right)$$

$$= \sin\left(-18\pi + \pi + \frac{\pi}{5}\right)$$

$$= \sin\left(\pi + \frac{\pi}{5}\right)$$

$$= -\sin\left(\frac{\pi}{5}\right)$$

$$\sin\left(-\frac{84\pi}{5}\right) = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

إذن

$$\cos\left(\frac{101\pi}{5}\right) = \cos\left(\frac{100\pi + \pi}{5}\right)$$

$$= \cos\left(20\pi + \frac{\pi}{5}\right)$$

$$= \cos\frac{\pi}{5}$$

$$\cos\left(\frac{101\pi}{5}\right) = \frac{\sqrt{5} + 1}{4}$$

إذن

تمرين 24

حل في \mathbb{R} المعادلات التالية :

$$* \cos x = \frac{\sqrt{2}}{2}$$

$$* 2\sin\left(x - \frac{\pi}{3}\right) = 1$$

$$* \sqrt{3} - 2\cos 2x = 0$$

$$* \tan\left(2x - \frac{\pi}{6}\right) = 1$$

$$= \cos\left(\frac{\pi}{2} + \frac{\pi}{5}\right)$$

$$= -\sin\frac{\pi}{5}$$

$$\cos\left(\frac{7\pi}{10}\right) = -\frac{\sqrt{5} + 1}{4}$$

$$\sin\left(\frac{-3\pi}{10}\right) = -\sin\left(\frac{3\pi}{10}\right)$$

$$= -\sin\left(\frac{5\pi - 2\pi}{10}\right)$$

$$= -\sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$$

$$= -\cos\frac{\pi}{5}$$

$$\sin\left(\frac{-3\pi}{10}\right) = -\frac{\sqrt{5} + 1}{4}$$

$$\tan\left(\frac{3\pi}{10}\right) = \frac{\sin\frac{3\pi}{10}}{\cos\frac{3\pi}{10}} = \frac{\cos\frac{\pi}{5}}{\sin\frac{\pi}{5}}$$

$$= \frac{\frac{\sqrt{5} + 1}{4}}{\frac{\sqrt{10 - 2\sqrt{5}}}{4}}$$

$$= \frac{\sqrt{5} + 1}{\sqrt{10 - 2\sqrt{5}}} = \frac{(\sqrt{5} + 1) \times \sqrt{10 + 2\sqrt{5}}}{\sqrt{10 - 2\sqrt{5}} \times \sqrt{10 + 2\sqrt{5}}}$$

$$= \frac{(\sqrt{5} + 1) \times \sqrt{2} \times \sqrt{5 + \sqrt{5}}}{\sqrt{80}}$$

$$= \frac{(\sqrt{5} + 1)\sqrt{2} \times \sqrt{5 + \sqrt{5}}}{4\sqrt{5}}$$

$$= \frac{(\sqrt{5} + 1)\sqrt{2} \times \sqrt{5} \times \sqrt{5 + \sqrt{5}}}{20}$$

$$= \frac{(\sqrt{5} + 1) \cdot \sqrt{2} \times \sqrt{5} \times \sqrt{5 + \sqrt{5}}}{20}$$

$$\cos 2x = \cos \frac{\pi}{6} \quad \text{إذن}$$

$$\begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ 2x = -\frac{\pi}{6} + 2k\pi \end{cases} \quad \text{أي أن } k \in \mathbf{Z}$$

$$\begin{cases} x = \frac{\pi}{12} + k\pi \\ x = -\frac{\pi}{12} + k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{أي أن}$$

$$S = \left\{ \frac{\pi}{12} + k\pi / k \in \mathbf{Z} \right\} \cup \left\{ -\frac{\pi}{12} + k\pi / k \in \mathbf{Z} \right\}$$

$$\tan\left(2x - \frac{\pi}{6}\right) = 1 \quad \text{لدينا}$$

$$\tan\left(2x - \frac{\pi}{6}\right) = \tan \frac{\pi}{4} \quad \text{أي أن}$$

$$2x - \frac{\pi}{6} = \frac{\pi}{4} + k\pi \quad \text{إذن}$$

$$k \in \mathbf{Z} \quad \text{مع}$$

$$2x = \frac{\pi}{4} + \frac{\pi}{6} + k\pi \quad \text{إذن}$$

$$k \in \mathbf{Z}, 2x = \frac{5\pi}{12} + k\pi$$

$$k \in \mathbf{Z}, x = \frac{5\pi}{24} + \frac{k\pi}{2} \quad \text{إذن}$$

$$S = \left\{ \frac{5\pi}{24} + \frac{k\pi}{2} / k \in \mathbf{Z} \right\}$$

تمرين 25:

حل في \mathbf{R} المعادلات التالية :

$$* 2 \cdot \cos 3x = -\sqrt{3}$$

$$* \sqrt{2} + 2 \sin\left(x - \frac{\pi}{4}\right) = 0$$

$$* \sqrt{3} + \tan 2x = 0$$

الجواب:

$$\cos x = \frac{\sqrt{2}}{2} \quad \text{لدينا}$$

$$\cos x = \cos \frac{\pi}{4} \quad \text{تكافئ}$$

$$\begin{cases} x = \frac{\pi}{4} + 2k\pi \\ x = -\frac{\pi}{4} + 2k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{أي أن}$$

إذن

$$S = \left\{ \frac{\pi}{4} + 2k\pi / k \in \mathbf{Z} \right\} \cup \left\{ -\frac{\pi}{4} + 2k\pi / k \in \mathbf{Z} \right\}$$

$$2 \sin\left(x - \frac{\pi}{3}\right) = 1 \quad \text{لدينا}$$

$$\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2} \quad \text{تكافئ}$$

$$\sin\left(x - \frac{\pi}{3}\right) = \sin \frac{\pi}{6} \quad \text{أي}$$

$$\begin{cases} x - \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi \\ x - \frac{\pi}{3} = \pi - \frac{\pi}{6} + 2k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{إذن}$$

$$\begin{cases} x = \frac{\pi}{3} + \frac{\pi}{6} + 2k\pi \\ x = \frac{5\pi}{6} + \frac{\pi}{3} + 2k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{إذن}$$

$$\begin{cases} x = \frac{\pi}{2} + 2k\pi \\ x = \frac{7\pi}{6} + 2k\pi \end{cases} \quad k \in \mathbf{Z} \quad \text{إذن}$$

إذن

$$S = \left\{ \frac{\pi}{2} + 2k\pi / k \in \mathbf{Z} \right\} \cup \left\{ \frac{7\pi}{6} + 2k\pi / k \in \mathbf{Z} \right\}$$

$$\sqrt{3} - 2 \cos 2x = 0 \quad \text{لدينا}$$

$$-2 \cos 2x = -\sqrt{3} \quad \text{أي أن}$$

$$\cos 2x = \frac{\sqrt{3}}{2}$$

$$x = 2k\pi \text{ أو } x = \frac{3\pi}{2} + 2k\pi$$

إذن

$$S = \{2k\pi / k \in \mathbb{Z}\} \cup \left\{ \frac{3\pi}{2} + 2k\pi / k \in \mathbb{Z} \right\}$$

$$\sqrt{3} + \tan 2x = 0 \quad \text{لدينا}$$

$$\tan 2x = -\sqrt{3} \quad \text{تكافئ}$$

$$\tan 2x = \tan\left(-\frac{\pi}{3}\right) \quad \text{أي أن}$$

$$k \in \mathbb{Z}, 2x = -\frac{\pi}{3} + k\pi \quad \text{أي أن}$$

$$x = -\frac{\pi}{6} + \frac{k\pi}{2} \quad \text{إذن}$$

$$S = \left\{ -\frac{\pi}{6} + \frac{k\pi}{2} / k \in \mathbb{Z} \right\} \quad \text{إذن}$$

تمرين 26:

1- حل في المجال $[0 ; 2\pi]$ المعادلة :

$$2\cos\left(x + \frac{\pi}{3}\right) = 1$$

2- حل في المجال $[-\pi ; 2\pi]$ المعادلة :

$$2\sin \frac{x}{2} = \sqrt{2}$$

3- حل في المجال $[-\pi ; \pi]$ المعادلة :

$$\cos^2 x + \cos x = 0$$

4- حل في المجال $[0 ; 3\pi]$ المعادلة :

$$\sin^2 x - 2\sin x = 0$$

الجواب:

$$2\cos\left(x + \frac{\pi}{3}\right) = 1 \quad \text{1- لدينا}$$

$$\cos\left(x + \frac{\pi}{3}\right) = \frac{1}{2} \quad \text{أي أن}$$

$$\cos\left(x + \frac{\pi}{3}\right) = \cos \frac{\pi}{3}$$

الجواب:

$$2.\cos 3x = -\sqrt{3} \quad \text{لدينا}$$

$$\cos 3x = -\frac{\sqrt{3}}{2} \quad \text{أي أن}$$

$$\cos 3x = -\cos \frac{\pi}{6} \quad \text{إذن}$$

$$\cos 3x = \cos\left(\pi - \frac{\pi}{6}\right) \quad \text{أي أن}$$

$$\cos 3x = \cos \frac{5\pi}{6} \quad \text{إذن}$$

$$\begin{cases} 3x = \frac{5\pi}{6} + 2k\pi \\ 3x = -\frac{5\pi}{6} + 2k\pi \end{cases} \quad \text{أو } k \in \mathbb{Z} \quad \text{أي أن}$$

$$\begin{cases} x = \frac{5\pi}{18} + \frac{2k\pi}{3} \\ x = -\frac{5\pi}{18} + \frac{2k\pi}{3} \end{cases} \quad \text{أي أن مع } k \in \mathbb{Z}$$

إذن

$$S = \left\{ \frac{5\pi}{18} + \frac{2k\pi}{3} / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{5\pi}{18} + \frac{2k\pi}{3} / k \in \mathbb{Z} \right\}$$

$$\sqrt{2} + 2\sin\left(x - \frac{\pi}{4}\right) = 0 \quad \text{لدينا}$$

$$2\sin\left(x - \frac{\pi}{4}\right) = -\sqrt{2} \quad \text{تكافئ}$$

$$\sin\left(x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \quad \text{أي أن}$$

$$\sin\left(x - \frac{\pi}{4}\right) = -\sin \frac{\pi}{4} \quad \text{أي أن}$$

$$\sin\left(x - \frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) \quad \text{إذن}$$

$$x - \frac{\pi}{4} = \pi + \frac{\pi}{4} + 2k\pi \quad \text{إذن}$$

$$k \in \mathbb{Z}, x - \frac{\pi}{4} = -\frac{\pi}{4} + 2k\pi \quad \text{أو}$$

$$\begin{cases} \frac{x}{2} = \frac{\pi}{4} + 2k\pi \\ \text{أو} \\ \frac{x}{2} = \pi - \frac{\pi}{4} + 2k\pi \end{cases} \quad k \in \mathbb{Z} \text{ أي أن}$$

$$\begin{cases} x = \frac{\pi}{2} + 4k\pi \\ \text{أو} \\ x = \frac{3\pi}{2} + 4k\pi \end{cases} \quad k \in \mathbb{Z} \text{ أي أن}$$

* لدينا $x \in [-\pi ; 2\pi]$ و $x = \frac{\pi}{2} + 4k\pi$

$$-\pi \leq \frac{\pi}{2} + 4k\pi \leq 2\pi \quad \text{إذن}$$

$$-1 \leq \frac{1}{2} + 4k \leq 2 \quad \text{أي أن}$$

$$-\frac{3}{2} \leq 4k \leq \frac{3}{2}$$

$$-\frac{3}{8} \leq k \leq \frac{3}{8} \quad \text{إذن}$$

وبما أن $k \in \mathbb{Z}$ فإن $k = 0$

$$x = \frac{\pi}{2} \quad \text{ومنه}$$

* لدينا $x \in [-\pi ; 2\pi]$ و $x = \frac{3\pi}{2} + 4k\pi$

$$-\pi \leq \frac{3\pi}{2} + 4k\pi \leq 2\pi \quad \text{إذن}$$

$$-1 \leq \frac{3}{2} + 4k \leq 2 \quad \text{أي أن}$$

$$-\frac{5}{2} \leq 4k \leq \frac{1}{2} \quad \text{ومنه}$$

$$-\frac{5}{8} \leq k \leq \frac{1}{8}$$

وبما أن $k \in \mathbb{Z}$ فإن $k = 0$

$$x = \frac{3\pi}{2} \quad \text{ومنه}$$

$$S = \left\{ \frac{3\pi}{2}, \frac{\pi}{2} \right\} \quad \text{إذن}$$

3- لدينا $\cos^2 x + \cos x = 0$

تكافئ $\cos x (\cos x + 1) = 0$

أي أن $\cos x = 0$ أو $\cos x + 1 = 0$

أي أن $\cos x = 0$ أو $\cos x = -1$

$$x + \frac{\pi}{3} = \frac{\pi}{3} + 2\pi k \quad \text{أي أن}$$

$$x + \frac{\pi}{3} = -\frac{\pi}{3} + 2\pi k \quad \text{أو}$$

حيث $k \in \mathbb{Z}$

$$\begin{cases} x = 2k\pi \\ \text{أو} \\ x = -\frac{2\pi}{3} + 2k\pi \end{cases} \quad k \in \mathbb{Z} \text{ أي أن}$$

* لدينا $x \in [0 ; 2\pi]$ و $x = 2\pi k$

$$0 \leq 2\pi k \leq 2\pi \quad \text{إذن}$$

$$0 \leq 2k \leq 2 \quad \text{أي أن}$$

$$0 \leq k \leq 1 \quad \text{ومنه}$$

وبما أن $k \in \mathbb{Z}$ فإن $k = 0$ أو $k = 1$

إذا كان $k = 0$ فإن $x = 0$

إذا كان $k = 1$ فإن $x = 2\pi$

* لدينا كذلك $x = -\frac{2\pi}{3} + 2k\pi$

و $x \in [0 ; 2\pi]$

$$0 \leq -\frac{2\pi}{3} + 2k\pi \leq 2\pi \quad \text{إذن}$$

$$0 \leq -\frac{2}{3} + 2k \leq 2 \quad \text{إذن}$$

$$\frac{2}{3} \leq 2k \leq \frac{8}{3} \quad \text{ومنه}$$

$$\frac{1}{3} \leq k \leq \frac{4}{3} \quad \text{إذن}$$

وبما أن $k \in \mathbb{Z}$ فإن $k = 1$ إذن

إذا كان $k = 1$ فإن $k = \frac{4\pi}{3}$

$$S = \left\{ 0 ; 2\pi, \frac{4\pi}{3} \right\} \quad \text{إذن}$$

2- لدينا $2\sin \frac{x}{2} = \sqrt{2}$

$$\sin \frac{x}{2} = \frac{\sqrt{2}}{2}$$

أي أن $\sin \frac{x}{2} = \sin \frac{\pi}{4}$

$$0 \leq k \leq 3$$

إذن $k = 0$ أو $k = 1$ أو $k = 2$ أو $k = 3$

إذا كان $k = 0$ فإن $x = 0$

إذا كان $k = 1$ فإن $x = \pi$

إذا كان $k = 2$ فإن $x = 2\pi$

إذا كان $k = 3$ فإن $x = 3\pi$

$$S = \{ 0 ; \pi ; 2\pi ; 3\pi \} \quad \text{إذن}$$

تمرين 27:

(1) - حل في المجال $[0, 2\pi]$

$$\cos(x) = \sin(x)$$

(2) - حل في المجال $[-\pi, 0]$

$$\cos\left(x - \frac{\pi}{3}\right) = -\sin(x)$$

(3) - حل في المجال $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\tan(x) = \sin(x)$$

الجواب:

(1) - لدينا $\cos(x) = \sin(x)$

$$\cos(x) = \cos\left(\frac{\pi}{2} - x\right) \quad \text{تكافئ}$$

$$x = \frac{\pi}{2} - x + 2k\pi \quad \text{أي أن}$$

$$k \in \mathbf{Z}, \quad x = -\frac{\pi}{2} + x + 2k\pi \quad \text{أو}$$

$$x + x = \frac{\pi}{2} + 2k\pi \quad \text{أي أن}$$

$$k \in \mathbf{Z}, \quad x - x = -\frac{\pi}{2} + 2k\pi \quad \text{أو}$$

$$2x = \frac{\pi}{2} + 2k\pi \quad \text{أي أن}$$

$$\text{إذن } x = \frac{\pi}{2} + k\pi \text{ أو } x = \pi + 2k\pi$$

مع $k \in \mathbf{Z}$

$$* \text{ لدينا } x \in [-\pi ; \pi] \text{ و } x = \frac{\pi}{2} + k\pi$$

$$\text{إذن } -\pi \leq \frac{\pi}{2} + k\pi \leq \pi$$

$$-1 \leq \frac{1}{2} + k \leq 1$$

$$-\frac{3}{2} \leq k \leq \frac{1}{2}$$

$k \in \mathbf{Z}$ إذن $k = -1$ أو $k = 0$

$$x = -\frac{\pi}{2} \quad \text{إذن } k = -1$$

$$x = \frac{\pi}{2} \quad \text{إذن } k = 0$$

$$* \text{ لدينا } x \in [-\pi ; \pi] \text{ و } x = \pi + 2k\pi$$

$$\text{إذن } -\pi \leq \pi + 2k\pi \leq \pi$$

$$\text{أي } -2 \leq 2k \leq 0$$

$$-1 \leq k \leq 0$$

$k \in \mathbf{Z}$ إذن $k = 0$ أو $k = -1$

إذا كان $k = 0$ فإن $x = \pi$

إذا كان $k = -1$ فإن $x = -\pi$

$$\text{إذن } S = \left\{ -\frac{\pi}{2} ; \frac{\pi}{2} ; \pi ; -\pi \right\}$$

$$4 - \text{ لدينا } \sin^2 x - 2\sin x = 0$$

$$\text{تكافئ } \sin x(\sin x - 2) = 0$$

$$\text{أي أن } \sin x = 0 \text{ أو } \sin x = 2$$

لا يمكن لأن $-1 \leq \sin x \leq 1$ أو $x = k\pi$ مع

$k \in \mathbf{Z}$

$$* \text{ لدينا } x \in [0 ; 3\pi] \text{ و } x = k\pi$$

$$\text{إذن } -0 \leq k\pi \leq 3\pi$$

$$-\frac{11}{12} \leq k \leq \frac{1}{12} \quad \text{إذن}$$

$$k = 0 \quad \text{بما أن } k \in \mathbf{Z} \quad \text{فإن}$$

$$S = \left\{ -\frac{11}{12} \right\} \quad \text{إذن}$$

$$(E) : \tan(x) = \sin(x) \quad \text{لدينا}$$

$$x \neq \frac{\pi}{2} + k\pi \quad \text{تكافئ } x \in D_E$$

$$\frac{\sin x}{\cos x} = \sin(x) \quad \text{لدينا (E) تكافئ}$$

$$\sin x = \cos x \cdot \sin x \quad \text{أي أن}$$

$$\sin x - \cos x \cdot \sin x = 0 \quad \text{أي أن}$$

$$\sin x(1 - \cos x) = 0 \quad \text{أي أن}$$

$$\sin x = 0 \quad \text{أو} \quad 1 - \cos x = 0 \quad \text{تكافئ}$$

$$\sin x = 0 \quad \text{أو} \quad \cos x = 1 \quad \text{ومنه}$$

$$x = k\pi \quad \text{أو} \quad x = 2k\pi \quad \text{أي أن}$$

$$k \in \mathbf{Z}, \quad x = k\pi \quad \text{إذن}$$

$$x = k\pi \quad \text{و} \quad x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\quad \text{بما أن}$$

$$-\frac{\pi}{2} \leq k\pi \leq \frac{\pi}{2} \quad \text{فإن}$$

$$-\frac{1}{2} \leq k \leq \frac{1}{2}$$

$$k = 0 \quad \text{بما أن } k \in \mathbf{Z} \quad \text{فإن}$$

$$x = 0 \quad \text{إذن}$$

$$S = \{0\} \quad \text{إذن}$$

$$0 = -\frac{\pi}{2} + 2k\pi \quad \text{لا يمكن}$$

$$x = \frac{\pi}{4} + k\pi \quad \text{إذن}$$

$$x \in [0, 2\pi] \quad \text{و} \quad x = \frac{\pi}{4} + k\pi \quad \text{لدينا}$$

$$0 \leq \frac{\pi}{4} + k\pi \leq 2\pi \quad \text{إذن}$$

$$0 \leq \frac{1}{4} + k \leq 2$$

$$-\frac{1}{4} \leq k \leq \frac{7}{4} \quad \text{أي أن}$$

$$k \in \mathbf{Z} \quad \text{فإن } k = 0 \quad \text{أو} \quad k = 1$$

$$x = \frac{\pi}{4} \quad \text{فإن } k = 0 \quad \text{إذا كان}$$

$$x = \frac{5\pi}{4} \quad \text{فإن } k = 1 \quad \text{إذا كان}$$

$$S = \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\} \quad \text{إذن}$$

$$\cos\left(x - \frac{\pi}{3}\right) = -\sin(x) \quad \text{(2) لدينا}$$

$$\cos\left(x - \frac{\pi}{3}\right) = \sin(-x) \quad \text{تكافئ}$$

$$\cos\left(x - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{2} + x\right) \quad \text{أي أن}$$

$$x - \frac{\pi}{3} = \frac{\pi}{2} + x + 2k\pi \quad \text{إذن}$$

$$x - \frac{\pi}{3} = -\frac{\pi}{2} - x + 2k\pi, \quad k \in \mathbf{Z} \quad \text{أو}$$

$$x - x = \frac{\pi}{2} + \frac{\pi}{3} + 2k\pi \quad \text{تكافئ}$$

$$x + x = -\frac{\pi}{2} + \frac{\pi}{3} + 2k\pi$$

$$0 = \frac{5\pi}{6} + 2k\pi, \quad k \in \mathbf{Z} \quad \text{لا يمكن}$$

$$2x = -\frac{\pi}{6} + 2k\pi \quad \text{إذن}$$

$$k \in \mathbf{Z}, \quad x = -\frac{\pi}{12} + k\pi \quad \text{إذن}$$

$$x = -\frac{\pi}{12} + k\pi \quad \text{و} \quad x \in [-\pi, 0] \quad \text{بما أن}$$

$$-\pi \leq -\frac{\pi}{12} + k\pi \leq 0 \quad \text{فإن}$$

$$-1 \leq -\frac{1}{12} + k \leq 0$$

تمرين 28:

$$(E') \quad 2\sin^2 x + (2 - \sqrt{2})\sin x - \sqrt{2} = 0$$

$$X = \sin x \quad \text{لنضع}$$

$$\text{إذن (E')} \quad 2X^2 + (2 - \sqrt{2})X - \sqrt{2} = 0 \quad \text{تكافئ}$$

$$\Delta = (2 - \sqrt{2})^2 - 4 \times 2 \times (-\sqrt{2})$$

$$= 2^2 - 4\sqrt{2} + \sqrt{2}^2 + 8\sqrt{2}$$

$$= 2^2 + 4\sqrt{2} + \sqrt{2}^2$$

$$= (2 + \sqrt{2})^2$$

$$\sqrt{\Delta} = 2 + \sqrt{2} \quad \text{إذن}$$

$$X = \frac{-2 + \sqrt{2} + 2 + \sqrt{2}}{4} \quad \text{أو} \quad X = \frac{-2 + \sqrt{2} - 2 - \sqrt{2}}{4}$$

$$X = \frac{\sqrt{2}}{2} \quad \text{أو} \quad X = -1 \quad \text{أي أن}$$

$$\sin x = \frac{\sqrt{2}}{2} \quad \text{أو} \quad \sin x = -1$$

$$\text{تكافئ} \quad \sin x = \sin \frac{\pi}{4} \quad \text{أو} \quad \sin x = -1$$

$$x = \frac{\pi}{4} + 2k\pi \quad \text{أو} \quad x = -\frac{\pi}{2} + 2k\pi$$

$$\text{أو} \quad x = \frac{3\pi}{4} + 2k\pi$$

إذن

$$S = \left\{ \frac{\pi}{4} + 2k\pi / k \in \mathbf{Z} \right\} \cup \left\{ \frac{3\pi}{4} + 2k\pi / k \in \mathbf{Z} \right\}$$

$$\cup \left\{ -\frac{\pi}{2} + 2k\pi / k \in \mathbf{Z} \right\}$$

(3) - لدينا

$$(E'') \quad \tan^2 x + (\sqrt{3} - 1)\tan x - \sqrt{3} = 0$$

$$k \in \mathbf{Z} \quad x \neq \frac{\pi}{2} + k\pi \quad \text{تكافئ} \quad x \in D(E'')$$

$$X = \tan x \quad \text{لنضع}$$

$$\text{إذن (E'')} \quad X^2 + (\sqrt{3} - 1)X - \sqrt{3} = 0 \quad \text{تكافئ}$$

(1) - حل في R المعادلة :

$$(E) \quad 2\cos^2 x - 5\cos x - 3 = 0$$

(2) - حل في R المعادلة :

$$(E') \quad 2\sin^2 x + (2 - \sqrt{2})\sin x - \sqrt{2} = 0$$

(3) - حل في R المعادلة :

$$(E'') \quad \tan^2 x + (\sqrt{3} - 1)\tan x - \sqrt{3} = 0$$

الجواب:

$$(1) - \text{لدينا} \quad 2\cos^2 x - 5\cos x - 3 = 0$$

$$X = \cos x \quad \text{لنضع}$$

$$\text{إذن (E)} \quad 2X^2 - 5X - 3 = 0 \quad \text{تكافئ}$$

$$\Delta = 25 + 24 = 49$$

$$\sqrt{\Delta} = 7 \quad \text{لدينا}$$

$$\text{إذن} \quad X = \frac{5+7}{4} \quad \text{أو} \quad X = \frac{5-7}{4}$$

$$\text{أي أن} \quad X = 3 \quad \text{أو} \quad X = -\frac{1}{2}$$

$$\text{أي أن} \quad \cos x = 3 \quad \text{أو} \quad \cos x = -\frac{1}{2}$$

$$\cos x = 3 \quad \text{لا يمكن}$$

$$\text{أو} \quad \cos x = \cos \frac{2\pi}{3}$$

$$\begin{cases} x = \frac{2\pi}{3} + 2k\pi \\ \text{أو} \\ x = -\frac{2\pi}{3} + 2k\pi \end{cases} \quad k \in \mathbf{Z}$$

$$S = \left\{ \frac{2\pi}{3} + 2k\pi / k \in \mathbf{Z} \right\} \cup \left\{ -\frac{2\pi}{3} + 2k\pi / k \in \mathbf{Z} \right\}$$

(2) - لدينا

الجواب :

$$(I) \begin{cases} x \in [0, 2\pi] \\ -1 + 2\cos x \geq 0 \end{cases}$$

نعتبر المعادلة : $-1 + 2\cos x = 0$

$$\cos x = \frac{1}{2} \quad \text{أي أن}$$

$$\cos x = \cos \frac{\pi}{3} \quad \text{إذن}$$

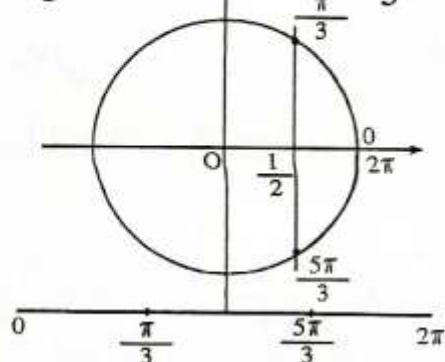
إذن :

$$x = \frac{\pi}{3} + 2k\pi \quad \text{أو} \quad x = -\frac{\pi}{3} + 2k\pi$$

حيث $k \in \mathbb{Z}$

وبما أن $x \in [0, 2\pi]$ فإن :

$$x = \frac{\pi}{3} \quad \text{أو} \quad x = \frac{5\pi}{3}$$



(I) تكافئ $\cos x \geq \frac{1}{2}$ أي أن

$$0 \leq x \leq \frac{\pi}{3} \quad \text{أو} \quad \frac{5\pi}{3} \leq x \leq 2\pi$$

$$S = \left[0, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right] \quad \text{إذن}$$

2 - لدينا

$$(I) \begin{cases} x \in [-\pi, \pi] \\ -2\sin x + \sqrt{2} < 0 \end{cases}$$

$$(I) \begin{cases} x \in [-\pi, \pi] \\ \sin x > \frac{\sqrt{2}}{2} \end{cases} \quad \text{تكافئ :}$$

$$\Delta = (\sqrt{3} - 1)^2 + 4\sqrt{3}$$

$$= \sqrt{3}^2 - 2\sqrt{3} + 1^2 + 4\sqrt{3}$$

$$= \sqrt{3}^2 + 2\sqrt{3} + 1^2$$

$$= (\sqrt{3} + 1)^2$$

$$\sqrt{\Delta} = \sqrt{3} + 1 \quad \text{إذن}$$

$$X = \frac{-\sqrt{3} + 1 + \sqrt{3} + 1}{2} = 1 \quad \text{ومنه}$$

$$X = \frac{-\sqrt{3} + 1 - \sqrt{3} - 1}{2} = -\sqrt{3} \quad \text{أو}$$

$$\tan x = 1 \quad \text{أو} \quad \tan x = -\sqrt{3} \quad \text{إذن}$$

$$\tan x = \tan \frac{\pi}{4} \quad \text{أو} \quad \tan x = -\tan \frac{\pi}{3}$$

$$\tan x = \tan \frac{\pi}{4} \quad \text{أو} \quad \tan x = \tan \left(-\frac{\pi}{3}\right)$$

$$x = \frac{\pi}{4} + k\pi \quad \text{ومنه}$$

$$\text{أو} \quad k \in \mathbb{Z}$$

$$x = -\frac{\pi}{3} + k\pi$$

$$S = \left\{ \frac{\pi}{4} + k\pi / k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{3} + k\pi / k \in \mathbb{Z} \right\}$$

تمرين 29 :

$$1 - \text{حل في المجال } [0, 2\pi]$$

$$\text{المراجعة : } -1 + 2\cos x \geq 0$$

$$2 - \text{حل في المجال } [0, 2\pi] \text{ المراجعة :}$$

$$-2\sin x + \sqrt{2} < 0$$

$$3 - \text{حل في المجال } \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[$$

$$\text{المراجعة : } \sqrt{3} - \tan x \leq 0$$

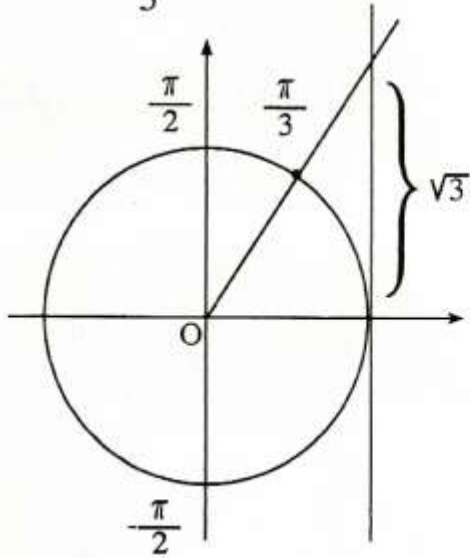
$\tan x = \sqrt{3}$: تكافئ

$\tan x = \tan \frac{\pi}{3}$ أي أن

$x = \frac{\pi}{3} + k\pi, k \in \mathbb{Z}$

بما أن $x \in]-\frac{\pi}{2}, \frac{\pi}{2}[$ فإن :

$x = \frac{\pi}{3}$



$\tan x \geq \sqrt{3}$ تكافئ (I')

$\frac{\pi}{3} \leq x < \frac{\pi}{2}$ أي أن

$S = [\frac{\pi}{3}, \frac{\pi}{2}[$ إذن

تمرين 30

1 - حل في المجال $[0, 2\pi]$

المترابحة : $2\cos x(2x) \geq \sqrt{3}$

2 - حل في المجال $[-\pi, \pi]$

المترابحة : $\sin(2x + \frac{\pi}{4}) < \frac{\sqrt{2}}{2}$

3 - حل في المجال $[0, \pi[$

$\tan \frac{x}{2} < 1$

نعتبر المعادلة : $\sin x = \frac{\sqrt{2}}{2}$

$\sin x = \frac{\pi}{4}$ أي أن

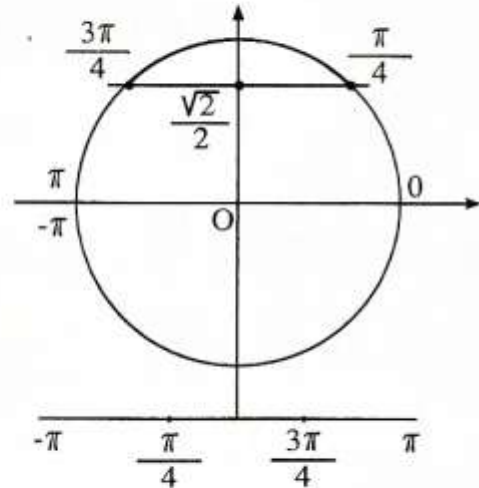
إذن :

$x = \frac{\pi}{4} + 2k\pi$ أو $x = \frac{3\pi}{4} + 2k\pi$

$k \in \mathbb{Z}$ حيث

وبما أن $x \in [-\pi, \pi]$ فإن :

$x = \frac{\pi}{4}$ أو $x = \frac{3\pi}{4}$



$\sin x > \frac{\sqrt{2}}{2}$ تكافئ (I')

$\frac{\pi}{4} < x < \frac{3\pi}{4}$ أي أن

$S =]\frac{\pi}{4}, \frac{3\pi}{4}[$ إذن

2 - لدينا

(I'') $\begin{cases} x \in]-\frac{\pi}{2}, \frac{\pi}{2}[\\ \sqrt{3} - \tan x \leq 0 \end{cases}$

نعتبر المعادلة : $\sqrt{3} - \tan x = 0$

إذن : $0 < x < \frac{\pi}{12}$ أو $\frac{11\pi}{12} < x < \pi$

إذن : $S = \left[\frac{\pi}{4}, \frac{3\pi}{4} \right] \cup \left[\frac{11\pi}{12}, \pi \right]$

2 - لدينا

$$(I) \begin{cases} x \in [-\pi, \pi] \\ \sin(2x + \frac{\pi}{4}) < \frac{\sqrt{2}}{2} \end{cases}$$

نعتبر المعادلة : $X = 2x + \frac{\pi}{4}$

لدينا : $-\pi \leq x \leq \pi$

أو $-2\pi \leq 2x \leq -2\pi$

أي أن : $-\frac{7\pi}{4} \leq 2x + \frac{\pi}{4} \leq \frac{9\pi}{4}$

$$(I) : \begin{cases} X \in \left[-\frac{7\pi}{4}, \frac{9\pi}{4} \right] \\ \sin X < \frac{\sqrt{2}}{2} \end{cases}$$

نعتبر المعادلة : $\sin X = \frac{\sqrt{2}}{2}$

$$\sin X = \sin \frac{\pi}{4}$$

$$X = \frac{\pi}{4} + 2k\pi \quad \text{أو} \quad X = \frac{3\pi}{4} + 2k\pi$$

مع $k \in \mathbb{Z}$

بما أن $X \in \left[-\frac{7\pi}{4}, \frac{9\pi}{4} \right]$ فإن :

$$X = \frac{\pi}{4} \quad \text{أو} \quad X = -\frac{7\pi}{4} \quad \text{أو} \quad X = \frac{9\pi}{4}$$

الجواب :

$$(I) : \begin{cases} x \in [0, \pi] \\ 2\cos x(2x) \geq \sqrt{3} \end{cases}$$

لنضع $X = 2x$

$$X \in [0, 2\pi] \quad x \in [0, \pi]$$

$$(I) \text{ تكافئ : } \begin{cases} X \in [0, 2\pi] \\ 2\cos X \geq \sqrt{3} \end{cases}$$

نعتبر المعادلة : $2\cos X = \sqrt{3}$

$$\cos X = \frac{\sqrt{3}}{2} \quad \text{أي أن}$$

$$\cos X = \cos \frac{\pi}{6} \quad \text{إذن}$$

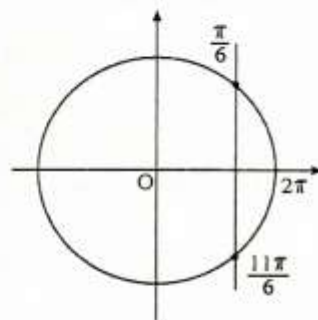
إذن :

$$X = \frac{\pi}{6} + 2k\pi \quad \text{أو} \quad X = -\frac{\pi}{6} + 2k\pi$$

مع $k \in \mathbb{Z}$

بما أن $X \in [0, 2\pi]$ فإن :

$$X = \frac{\pi}{6} \quad \text{أو} \quad X = \frac{11\pi}{6}$$



$$\sin x \geq \frac{\sqrt{3}}{2} \quad (I') \text{ تكافئ}$$

أي أن : $0 < X < \frac{\pi}{6}$ أو $\frac{11\pi}{6} < X < 2\pi$

أي أن : $0 < 2x < \frac{\pi}{6}$ أو $\frac{11\pi}{6} < 2x < 2\pi$

$$X = \frac{x}{2} \quad \text{لنضع}$$

$$X \in \left[0, \frac{\pi}{2}\right] \quad \text{إذن} \quad x \in [0, \pi]$$

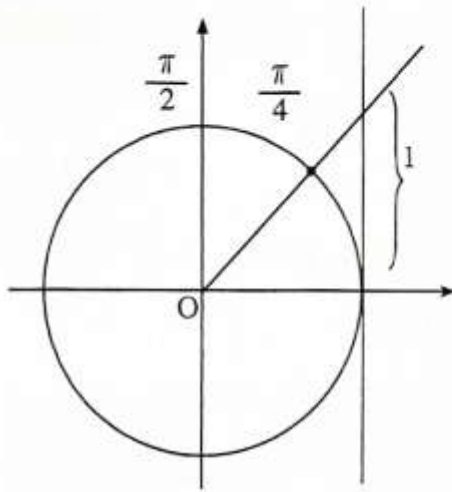
$$\begin{cases} X \in \left[0, \frac{\pi}{2}\right] \\ \tan X < 1 \end{cases} \quad \text{تكافئ (I')}$$

$$\tan X = 1 \quad \text{نعتبر المعادلة:}$$

$$\tan X = \tan \frac{\pi}{4}$$

$$k \in \mathbf{Z}, \quad X = \frac{\pi}{4} + k\pi$$

$$X = \frac{\pi}{4} \quad \text{وبما أن} \quad X \in \left[0, \frac{\pi}{2}\right] \quad \text{فإن:}$$



$$\tan X < 1 \quad \text{تكافئ (I'')} \quad \text{أي أن:}$$

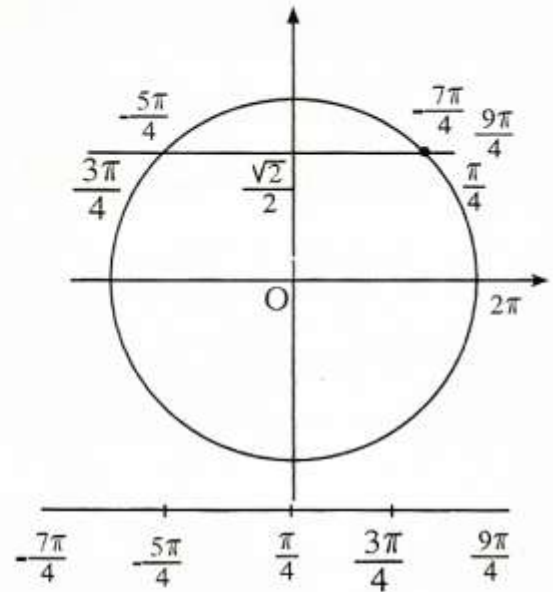
$$0 \leq X < \frac{\pi}{4} \quad \text{أي أن:}$$

$$0 \leq \frac{x}{2} < \frac{\pi}{4} \quad \text{أي أن:}$$

$$0 \leq x < \frac{\pi}{2} \quad \text{أي أن:}$$

$$S = \left[0, \frac{\pi}{2}\right[\quad \text{إذن:}$$

$$X = \frac{3\pi}{4} \quad \text{أو} \quad X = -\frac{5\pi}{4}$$



$$\sin X < \frac{\sqrt{2}}{2} \quad \text{تكافئ (I')}$$

$$-\frac{5\pi}{4} < X < \frac{\pi}{4} \quad \text{أو} \quad \frac{3\pi}{4} < X < \frac{9\pi}{4}$$

$$\text{أي أن:} \quad \frac{3\pi}{4} < 2x + \frac{\pi}{4} < \frac{9\pi}{4}$$

$$\text{أو} \quad -\frac{5\pi}{4} < 2x + \frac{\pi}{4} < \frac{\pi}{4}$$

$$\text{أي أن:} \quad \frac{\pi}{2} < 2x < 2\pi \quad \text{أو} \quad -\frac{3\pi}{2} < 2x < 0$$

$$\text{أي أن:} \quad \frac{\pi}{4} < x < \pi \quad \text{أو} \quad -\frac{3\pi}{2} < x < 0$$

$$\text{إذن:} \quad S = \left[-\frac{3\pi}{2}, 0\right] \cup \left[\frac{\pi}{4}, \pi\right]$$

$$(I'') : \quad \begin{cases} x \in [0, \pi] \\ \sin x \left(\frac{x}{2}\right) < 1 \end{cases} \quad \text{لدينا} \quad -3$$

تمرين 31:

ليكن لكل x من \mathbb{R}

$$Q(x) = -2\sin^2 x + 3\sin x - 1$$

1 - أ - بين أن لكل x من \mathbb{R} .

$$Q(x) = (2\sin x - 1)(1 - \sin x)$$

ب - حل في \mathbb{R} المعادلة $Q(x) = 0$

2 - أ - ادرس إشارة $Q(x)$ على $[-\pi, \pi]$

المجال

ب - استنتج حلول المتراجحة $Q(x) > 0$

على المجال $[-\pi, \pi]$

الجواب:

1 - أ - لدينا $(2\sin x - 1)(1 - 2\sin x)$

$$= 2\sin x - 2\sin^2 x - 1 + \sin x$$

$$= -2\sin^2 x + 3\sin x - 1$$

$$= Q(x)$$

إذن : $Q(x) = -2\sin^2 x + 3\sin x - 1$

ب - $Q(x) = 0$ تكافئ

$$(2\sin x - 1)(1 - \sin x) = 0$$

أي أن $2\sin x - 1 = 0$ أو $1 - \sin x = 0$

أي أن $\sin x = \frac{1}{2}$ أو $\sin x = 1$

$\sin x = \sin \frac{\pi}{6}$ أو $\sin x = 1$

$$x = \frac{\pi}{6} + 2k\pi \text{ أو } x = \frac{5\pi}{6} + 2k\pi$$

$$x = \frac{\pi}{2} + 2k\pi$$

حيث $k \in \mathbb{Z}$

$$S = \left\{ \frac{\pi}{6} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{6} + 2k\pi / k \in \mathbb{Z} \right\}$$

$$\cup \left\{ \frac{\pi}{2} + 2k\pi / k \in \mathbb{Z} \right\}$$

2 - أ - حلول المعادلة $Q(x) = 0$ على

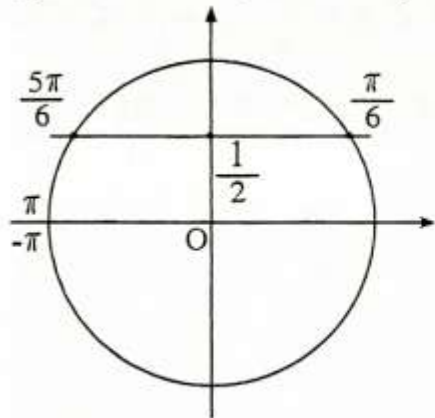
المجال $[-\pi, \pi]$ هي :

$$x = \frac{\pi}{6} \text{ أو } x = \frac{5\pi}{6} \text{ أو } x = \frac{\pi}{2}$$

x	$-\pi$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π
$1 - \sin x$	+	+	○	+	+
$2\sin x - 1$	-	○	+	+	○
$Q(x)$	-	○	+	○	-

ملاحظة : $1 \leq \sin x \leq 1$

إذن $\sin x \leq 1$ أي أن $0 \leq 1 - \sin x$



$$\begin{cases} x \in [-\pi, \pi] \\ Q(x) > 1 \end{cases} \quad \text{ب -}$$

تكافئ : $\frac{\pi}{2} < x < \frac{5\pi}{6}$ أو $\frac{\pi}{6} < x < \frac{\pi}{2}$

$$S = \left] \frac{\pi}{6}, \frac{\pi}{2} \right[\cup \left] \frac{\pi}{2}, \frac{5\pi}{6} \right[\quad \text{إذن}$$

$$\in \mathbf{Z} \} \cup \left\{ -\frac{2\pi}{3} + 2k\pi / k \in \mathbf{Z} \right\}$$

2 - أ - تكافئ $P(x) = 0$

$$x = \frac{\pi}{2} + k\pi \quad \text{أو} \quad x = \frac{2\pi}{3} + 2k\pi$$

$$x = -\frac{2\pi}{3} + 2k\pi \quad \text{أو}$$

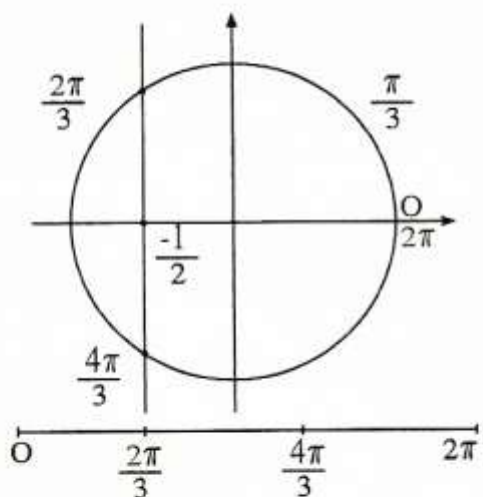
حيث $k \in \mathbf{Z}$

حلول المعادلة $P(x) = 0$ على المجال :

$[0, 2\pi]$ هي :

$$x = \frac{\pi}{2} \quad \text{أو} \quad x = \frac{3\pi}{2} \quad \text{أو} \quad x = \frac{2\pi}{3} \quad \text{أو} \quad x = \frac{4\pi}{3}$$

x	0	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	2π	
cosx	+	0	-	-	-	0	+
2cosx+1	+	+	0	-	0	+	+
P(x)	+	0	-	0	+	0	+



$P(x) \leq 0$ تكافئ

$$\frac{\pi}{2} \leq x \leq \frac{2\pi}{3} \quad \text{أو} \quad \frac{4\pi}{3} \leq x \leq \frac{3\pi}{2}$$

$$S = \left[\frac{\pi}{2}, \frac{2\pi}{3} \right] \cup \left[\frac{4\pi}{3}, \frac{3\pi}{2} \right] \quad \text{إذن}$$

تمرين 32

نعتبر لكل x من \mathbf{R}

$$P(x) = 2\cos^2 x + \cos x$$

1 - حل في \mathbf{R} المعادلة : $P(x) = 0$

2 - أ - أدرس إشارة $P(x)$ لكل x من

المجال $[0, 2\pi]$

ب - استنتج حلول المتراجحة $P(x) \leq 0$

لكل x من $[0, 2\pi]$

الجواب :

1 - $P(x) = 0$ تكافئ

$$2\cos^2 x + \cos x = 0$$

$$\cos x(2\cos x + 1) = 0$$

$$\cos x = 0 \quad \text{أو} \quad 2\cos x + 1 = 0$$

$$\cos x = 0 \quad \text{أو} \quad \cos x = -\frac{1}{2}$$

$$\cos x = 0 \quad \text{أو} \quad \cos x = -\cos \frac{\pi}{3}$$

$$\cos x = 0 \quad \text{أو} \quad \cos x = \cos \frac{2\pi}{3}$$

$$x = \frac{\pi}{2} + k\pi \quad \text{أو} \quad x = \frac{2\pi}{3} + 2k\pi$$

$$x = -\frac{2\pi}{3} + 2k\pi \quad \text{أو}$$

حيث $k \in \mathbf{Z}$

$$S = \left\{ \frac{\pi}{2} + k\pi / k \in \mathbf{Z} \right\} \cup \left\{ \frac{2\pi}{3} + 2k\pi / k \right\}$$