السنة 2 بكالوريا علوم تجريبية	الأعداد العقدية	سلسلة 2

تمرين 1 : اكتب الأعداد التالية على شكلها المثلثي.

$$z_1=3+3\boldsymbol{i}=3(1+\boldsymbol{i})=3\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\boldsymbol{i}\right)=3\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+\boldsymbol{i}\sin\left(\frac{\pi}{4}\right)\right)=\left[3\sqrt{2};\frac{\pi}{4}\right]$$

$$z_2=1-\boldsymbol{i}=\sqrt{2}\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\boldsymbol{i}\right)=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+\boldsymbol{i}\sin\left(-\frac{\pi}{4}\right)\right)=\left[\sqrt{2}\,;-\frac{\pi}{4}\right]$$

$$z_3=-\sqrt{3}-\boldsymbol{i}=2\left(\frac{-\sqrt{3}}{2}-\frac{1}{2}\boldsymbol{i}\right)=2\left(\cos\left(\frac{7\pi}{6}\right)+\boldsymbol{i}\sin\left(\frac{7\pi}{6}\right)\right)=\left[2\,;\frac{7\pi}{6}\right]$$

$$z_4=-\sqrt{2}+\sqrt{6}\,\boldsymbol{i}=2\sqrt{2}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\boldsymbol{i}\right)=2\sqrt{2}\left(\cos\left(\frac{2\pi}{3}\right)+\boldsymbol{i}\sin\left(\frac{2\pi}{3}\right)\right)=\left[2\sqrt{2}\,;\frac{\pi}{3}\right]$$

$$z_5=13=13\times1=(\cos(0)+\boldsymbol{i}\sin(0))=[13;0]$$

$$z_6=-7=7\times(-1)=(\cos(\pi)+\boldsymbol{i}\sin\pi)=[7;\pi]$$

$$z_7=11\,\boldsymbol{i}=11\times\boldsymbol{i}=\left(\cos\left(\frac{\pi}{2}\right)+\boldsymbol{i}\sin\left(\frac{\pi}{2}\right)\right)=\left[11;\frac{\pi}{2}\right]$$

$$z_8=\frac{-\boldsymbol{i}}{4}=\frac{1}{4}\times(-\boldsymbol{i})=\frac{1}{4}\left(\cos\left(\frac{-\pi}{2}\right)+\boldsymbol{i}\sin\left(\frac{-\pi}{2}\right)\right)=\left[\frac{1}{4};\frac{-\pi}{2}\right]$$

$$z_9=\frac{1}{7}+\frac{1}{7}\boldsymbol{i}=\frac{1}{7}(1+\boldsymbol{i})=\frac{\sqrt{2}}{7}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\boldsymbol{i}\right)=\left[\frac{\sqrt{2}}{7}\,;\frac{\pi}{4}\right]$$

$$z_{10}=\cos(\theta)-\boldsymbol{i}\sin(\theta)=\cos(-\theta)+\boldsymbol{i}\sin(-\theta)=[1;-\theta]$$

$$z_{11}=-\cos(\theta)-\boldsymbol{i}\sin(\theta)=\cos(\theta+\pi)+\boldsymbol{i}\sin(\theta+\pi)=[1;\theta+\pi]$$

$$z_{12}=\sin(\theta)+\boldsymbol{i}\cos(\theta)=\cos\left(\frac{\pi}{2}-\theta\right)+\boldsymbol{i}\sin\left(\frac{\pi}{2}-\theta\right)=\left[1;\frac{\pi}{2}-\theta\right]$$

$$z_{13}=1-\cos(2\ \ )+\boldsymbol{i}\sin(2\ \ )=2\sin^2(\ \ )+2\sin(\ \ )\cos(\ \ )\boldsymbol{i}=2\sin(\ \ )(\sin(\ \ )+\boldsymbol{i}\cos(\ \ ))$$

$$z_{13}=2\sin(\ \ )\left(\cos\left(\frac{\pi}{2}-\ \ \right)+\boldsymbol{i}\sin\left(\frac{\pi}{2}-\ \ \right)\right)=\left[2\sin(\ \ );\frac{\pi}{2}-\ \ \right]$$

(لأن: $2\sin(\ \ )>0 \Rightarrow \ \ \in\left]0,\frac{\pi}{2}\right[$)

للتذكير،للحصول على الشكل المثلثي للعدد : $z=a+\boldsymbol{i}b$ نعمل بمعيار العدد z أي نكتب: $z=r\left(\frac{a}{r}+\boldsymbol{i}\frac{b}{r}\right)$

حيث $r=\sqrt{a^2+b^2}$ ثم نبحث في جدول القيم الهامة عن الزاوية θ التي تحقق : $\cos(\theta)=\frac{a}{r}$ و $\sin(\theta)=\frac{b}{r}$

ليس من الضروري اتباع الطريقة السابقة في كل الحالات، فمثلا إن تبين لنا عامل مشترك نعمل به أولا ثم نطبق الطريقة على العامل المحصل عليه (مثل z_1 و z_5)

حالات خاصة : $a=[a,0]\,;\ -a=[a,\pi]\,;\ a\boldsymbol{i}=\left[a,\frac{\pi}{2}\right];\ -a\boldsymbol{i}=\left[a,-\frac{\pi}{2}\right]$ $\quad\forall a\in IR^{+*}$

يمكن استعمال خاصيات الكتابة المثلثية أحيانا لتحديد الشكل المثلثي لعدد عقدي، مثلا :

$$z_3=-\sqrt{3}-\boldsymbol{i}=-(\sqrt{3}+\boldsymbol{i})=-2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\boldsymbol{i}\right)=[-2\,;\pi\,]\times\left[1\,;\frac{\pi}{6}\right]=[-2\,;\pi\,]\times\left[2\times1\,;\pi+\frac{\pi}{6}\right]=\left[2;\frac{5\pi}{6}\right]$$

تمرين 2 :

1

$$u=\frac{\sqrt{6}+i\sqrt{2}}{2}=\frac{1}{2}\left(\sqrt{6}+i\sqrt{2}\right)=\frac{1}{2}2\sqrt{2}\left(\frac{\sqrt{6}}{2\sqrt{2}}+i\frac{\sqrt{2}}{2\sqrt{2}}\right)=\sqrt{2}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=\left[\sqrt{2}\,;\frac{\pi}{6}\right]$$

$$v=1-i=\sqrt{2}\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)=\left[\sqrt{2};\frac{-\pi}{4}\right]$$

2

$$z_1=uv=\left[\sqrt{2}\times\sqrt{2};\frac{\pi}{6}+\left(\frac{-\pi}{4}\right)\right]=\left[2\,;\frac{-\pi}{12}\right]$$

$$z_2=\frac{u}{v}=\left[\frac{\sqrt{2}}{\sqrt{2}};\frac{\pi}{6}-\left(\frac{-\pi}{4}\right)\right]=\left[1\,;\frac{5\pi}{12}\right]$$

$$z_3=u^5=\left[\sqrt{2}^5;5\times\frac{\pi}{6}\right]=\left[4\sqrt{2}\,;\frac{5\pi}{6}\right]$$

$$z_4=\frac{1}{v^7}=\left[\frac{1}{\sqrt{2}^7};-7\times\frac{-\pi}{4}\right]=\left[\frac{1}{8\sqrt{2}}\,;\frac{7\pi}{4}\right]$$

$$z_5=\frac{v^2}{u^3}=\left[\frac{\sqrt{2}^2}{\sqrt{2}^3};2\times\left(\frac{-\pi}{4}\right)-3\times\frac{\pi}{6}\right]=\left[\frac{\sqrt{2}}{2}\,;-\pi\right]$$

$$z_6=5u=[5;0]\times\left[\sqrt{2};\frac{\pi}{6}\right]=\left[5\sqrt{2};\frac{\pi}{6}\right]$$

$$z_7=-iv=\left[1;\frac{-\pi}{2}\right]\times\left[\sqrt{2};\frac{-\pi}{4}\right]=\left[1\times\sqrt{2};\frac{-\pi}{2}+\frac{-\pi}{4}\right]=\left[\sqrt{2};\frac{-3\pi}{4}\right]$$

تمرين 3 : المستوى العقدي منسوب إلى م.م.م $(O,\vec{e_1},\vec{e_2})$. $A(1+3i)$ ، $B\left(\frac{2-\sqrt{3}}{2}+\frac{5}{2}i\right)$ ، $C(1+2i)$

1

طريقة 1:

$$\frac{c-a}{b-a}=\frac{1+2i-(1+3i)}{\frac{2-\sqrt{3}}{2}+\frac{5}{2}i-(1+3i)}=\frac{1+2i-1-3i}{\frac{2-\sqrt{3}}{2}+\frac{5}{2}i-1-3i}=\frac{-i}{\frac{-\sqrt{3}}{2}-\frac{1}{2}i}=\frac{i}{\frac{\sqrt{3}}{2}+\frac{1}{2}i}$$

$$\frac{c-a}{b-a}=\frac{i\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)}{\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)}=\frac{\frac{\sqrt{3}}{2}i+\frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{1}{2}i\right)^2}=\frac{\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{3}{4}+\frac{1}{4}}=\frac{\frac{1}{2}+\frac{\sqrt{3}}{2}i}{1}=\frac{1}{2}+\frac{\sqrt{3}}{2}i=\left[1;\frac{\pi}{3}\right]$$

طريقة 2:

$$\frac{c-a}{b-a}=\frac{1+2i-(1+3i)}{\frac{2-\sqrt{3}}{2}+\frac{5}{2}i-(1+3i)}=\frac{1+2i-1-3i}{\frac{2-\sqrt{3}}{2}+\frac{5}{2}i-1-3i}=\frac{-i}{\frac{-\sqrt{3}}{2}-\frac{1}{2}i}=\frac{i}{\frac{\sqrt{3}}{2}+\frac{1}{2}i}$$

$$\frac{c-a}{b-a}=\frac{\left[1;\frac{\pi}{2}\right]}{\left[1;\frac{\pi}{6}\right]}=\left[1;\frac{\pi}{2}-\frac{\pi}{6}\right]=\left[1;\frac{\pi}{3}\right]$$

2

لدينا : $\frac{c-a}{b-a}=\left[1;\frac{\pi}{3}\right]$ منه : $\arg\left(\frac{c-a}{b-a}\right)\equiv\frac{\pi}{3}[2\pi]$ و $\left|\frac{c-a}{b-a}\right|=1$

إذن : $\left(\overrightarrow{AB},\overrightarrow{AC}\right)\equiv\arg\left(\frac{c-a}{b-a}\right)\equiv\frac{\pi}{3}[2\pi]$ و $\frac{|c-a|}{|b-a|}=1$ أي $|b-a|=|c-a|$ منه : $AB=AC$

3

بما أن $\left(\overrightarrow{AB},\overrightarrow{AC}\right)\equiv\frac{\pi}{3}[2\pi]$ و $AB=AC$ فالمثلث ABC متساوي الأضلاع.

تمرين 4 : $u=1+i\sqrt{3}$ ، $v=1+i$ ، $w=\frac{u}{v}$

1

$$w=\frac{u}{v}=\frac{1+i\sqrt{3}}{1+i}=\frac{(1+i\sqrt{3})((1-i))}{(1+i)(1-i)}=\frac{1-i+i\sqrt{3}-i^2\sqrt{3}}{1-i^2}=$$

$$w=\frac{1+i(-1+\sqrt{3})+\sqrt{3}}{1+1}=\frac{1+\sqrt{3}+i(\sqrt{3}-1)}{2}=\frac{1+\sqrt{3}}{2}+i\frac{\sqrt{3}-1}{2}$$

2

لدينا : $u=1+i\sqrt{3}=2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=\left[2;\frac{\pi}{3}\right]$ و $v=1+i=\sqrt{2}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=\left[\sqrt{2};\frac{\pi}{4}\right]$

منه : $w=\frac{u}{v}=\frac{\left[2;\frac{\pi}{3}\right]}{\left[\sqrt{2};\frac{\pi}{4}\right]}=\left[\frac{2}{\sqrt{2}};\frac{\pi}{3}-\frac{\pi}{4}\right]=\left[\sqrt{2};\frac{\pi}{12}\right]$

3

لدينا حسب السؤال 2 : $w=\left[\sqrt{2};\frac{\pi}{12}\right]=\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)=\sqrt{2}\cos\frac{\pi}{12}+i\sqrt{2}\sin\frac{\pi}{12}$

و حسب السؤال الأول : $w=\frac{1+\sqrt{3}}{2}+i\frac{\sqrt{3}-1}{2}$

إذن : $\begin{cases}\sqrt{2}\cos\frac{\pi}{12}=\frac{\sqrt{3}+1}{2}\\ \sqrt{2}\sin\frac{\pi}{12}=\frac{\sqrt{3}-1}{2}\end{cases}$ بالتالي : $\begin{cases}\cos\frac{\pi}{12}=\frac{\sqrt{3}+1}{2\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}\\ \sin\frac{\pi}{12}=\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{4}\end{cases}$

و منه : $\tan\frac{\pi}{12}=\frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}}=\frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}=\frac{\sqrt{3}+1}{\sqrt{3}-1}=\frac{(\sqrt{3}+1)^2}{3-1}=\frac{4+2\sqrt{3}}{2}=2+\sqrt{3}$

تمرين 5 : $Z=\sqrt{6}+\sqrt{2}+i(\sqrt{6}-\sqrt{2})$

1

$$Z^2=\left(\sqrt{6}+\sqrt{2}+i(\sqrt{6}-\sqrt{2})\right)^2=(\sqrt{6}+\sqrt{2})^2+2i(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})+i^2(\sqrt{6}-\sqrt{2})^2$$

$$Z^2=8+2\sqrt{12}+8i-(8-2\sqrt{12})=4\sqrt{12}+8i=8\sqrt{3}+8i$$

2

$$Z^2=8\sqrt{3}+8i=8(\sqrt{3}+i)=8\times2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=\left[16;\frac{\pi}{6}\right]$$

3

نضع : $Z=[r;\theta]$ حيث : $r\in IR^{+*}$ و $\theta\in]-0;2\pi]$ إذن : $Z^2=[r^2;2\theta]$

نستنتج إذن أن: $r^2=16$ منه : $r=4$ و أن $2\theta\equiv\frac{\pi}{6}+2k\pi\,/\,k\in Z$ أي : $\theta\equiv\frac{\pi}{12}+k\pi\,/\,k\in Z$

و بما أن : $\theta\in]0;2\pi]$ فإن : $0<\frac{\pi}{12}+k\pi\le2\pi$

منه : $\frac{-\pi}{12}<k\pi\le\frac{23\pi}{12}$ منه : $\frac{-1}{12}<k\le\frac{23}{12}$ منه : $k=0$ أو $k=1$ ، منه : $\theta=\frac{\pi}{12}$ أو $\theta=\frac{13\pi}{12}$

لكن و لكون : $\mathrm{Re}(Z)>0$ فإن : $\cos\theta>0$ ، إذن : $\theta=\frac{\pi}{12}$ (لأن : $\cos\left(\frac{13\pi}{12}\right)<0\Rightarrow\pi<\frac{13\pi}{12}<\frac{3\pi}{2}$

بالتالي : $Z=\left[4;\frac{\pi}{12}\right]$

تحديد الشكل المثلثي لعدد عقدي انطلاقا من الشكل المثلثي لمربعه أو مكعبه أو بصفة عامة قوة له يتطلب تقنيات خاصة حيث نعطي رموزا لمعياره و عمدته، ثم نحاول تحديدهما مستعملين وحدانية الشكل المثلثي و الجبري ، و نحتاج خلال ذلك للتأطير لحصر قيم العدد k و بالتالي حصر قيم العمدة ثم تحديده انطلاقا من إشارة الجزء الحقيقي أو التخيلي

4

لدينا حسب السؤال 2 : $Z = \left[4\,;\frac{\pi}{12}\right] = 4\left(\cos\frac{\pi}{12} + \boldsymbol{i}\sin\frac{\pi}{12}\right) = 4\cos\frac{\pi}{12} + 4\sin\frac{\pi}{12}\,\boldsymbol{i}$

و حيث أن : $Z = \sqrt{6}+\sqrt{2}+\boldsymbol{i}\left(\sqrt{6}-\sqrt{2}\right)$ فإن : $\begin{cases} 4\cos\frac{\pi}{12} = \sqrt{6}+\sqrt{2} \\ 4\sin\frac{\pi}{12} = \sqrt{6}-\sqrt{2} \end{cases}$ بالتالي : $\begin{cases} \cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4} \\ \sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4} \end{cases}$

و منه : $\tan\frac{\pi}{12} = \frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}} = \frac{\frac{\sqrt{6}+\sqrt{2}}{2}}{\frac{\sqrt{6}-\sqrt{2}}{2}} = \frac{\left(\sqrt{6}+\sqrt{2}\right)^2}{6-2} = \frac{8+2\sqrt{12}}{4} = 2+\sqrt{3}$

لاحظ أن نتائج هذا التمرين تطابق نتائج التمرين السابق