السنة 2 بكالوريا علوم رياضية	النهايات والاتصال حل مقترح	سلسلة 1

تمرين 1 :

$$\lim_{x\mapsto+\infty} x+3-\sqrt{x+3}=\lim_{x\mapsto+\infty}\left(\sqrt{x+3}\right)^2-\sqrt{x+3}=\lim_{x\mapsto+\infty}\sqrt{x+3}\left(\sqrt{x+3}-1\right)=+\infty$$

$$\lim_{x\mapsto+\infty} x+3-\sqrt{x^2+4x}=\lim_{x\mapsto+\infty} x+2-\sqrt{x^2+4x}+1=\lim_{x\mapsto+\infty}\frac{x^2+4x+4-x^2-4x}{x+2+\sqrt{x^2+4x}}+1$$

$$=\lim_{x\mapsto+\infty}\frac{4}{x+2+\sqrt{x^2+4x}}+1=0+1=1$$

$$\lim_{x\mapsto+\infty}\sqrt{\sqrt{x}+1}-\sqrt{x+\sqrt{x}}=\lim_{x\mapsto+\infty}\sqrt{\sqrt{x}\left(1+\frac{1}{\sqrt{x}}\right)}-\sqrt{x\left(1+\frac{\sqrt{x}}{x}\right)}$$

$$=\lim_{x\mapsto+\infty}\sqrt{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}-\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}=\lim_{x\mapsto+\infty}\sqrt{x}\left(\frac{\sqrt{\sqrt{x}}}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}-\sqrt{1+\frac{1}{\sqrt{x}}}\right)$$

$$=\lim_{x\mapsto+\infty}\sqrt{x}\left(\frac{1}{\sqrt{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}-\sqrt{1+\frac{1}{\sqrt{x}}}\right)=-\infty$$

$$\lim_{x\mapsto+\infty}\sqrt{1+2x^3}-\sqrt{x^3+x+1}=\lim_{x\mapsto+\infty}\sqrt{x^3}\sqrt{\frac{1}{x^3}+2}-\sqrt{x^3}\sqrt{1+\frac{1}{x^2}+\frac{1}{x^3}}$$

$$=\lim_{x\mapsto+\infty}\sqrt{x^3}\left(\sqrt{\frac{1}{x^3}+2}-\sqrt{1+\frac{1}{x^2}+\frac{1}{x^3}}\right)=+\infty$$

- إذا كان $m>0$ فإن : $\lim_{x\mapsto-\infty}\left(\sqrt{5x^2+x+1}-mx\right)=+\infty$ ($\lim_{x\mapsto-\infty}-mx=+\infty$)
- إذا كان $m=0$ فإن : $\lim_{x\mapsto-\infty}\left(\sqrt{5x^2+x+1}-mx\right)=\lim_{x\mapsto-\infty}\left(\sqrt{5x^2+x+1}\right)+\infty$
- إذا كان $-\sqrt{5}<m<0$ فإن :

$$\lim_{x\mapsto-\infty}\left(\sqrt{5x^2+x+1}-mx\right)=\lim_{x\mapsto-\infty}\left(\sqrt{x^2\left(5+\frac{1}{x}+\frac{1}{x^2}\right)}-mx\right)=\lim_{x\mapsto-\infty}\left(-x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-mx\right)$$

$$=\lim_{x\mapsto-\infty}-x\left(\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}+m\right)=+\infty$$

- إذا كان $m=-\sqrt{5}$ فإن :

$$\lim_{x\mapsto-\infty}\left(\sqrt{5x^2+x+1}-mx\right)=\lim_{x\mapsto-\infty}\sqrt{5x^2+x+1}+\sqrt{5}\,x=\lim_{x\mapsto-\infty}\frac{x+1}{\sqrt{5x^2+x+1}-\sqrt{5}\,x}$$

$$=\lim_{x\mapsto-\infty}\frac{x\left(1+\frac{1}{x}\right)}{-x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{5}\,x}=\lim_{x\mapsto-\infty}\frac{1+\frac{1}{x}}{-x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{5}\,x}=\frac{-1}{2\sqrt{5}}$$

- إذا كان $m<-\sqrt{5}$ فإن : $\lim_{x\mapsto-\infty}\left(\sqrt{5x^2+x+1}-mx\right)=\lim_{x\mapsto-\infty}-x\left(\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}+m\right)=-\infty$

$$\lim_{x\mapsto+\infty}\left(\frac{x-\sqrt{x^2+1}}{x^2-\sqrt{x^4-1}}\right)=\lim_{x\mapsto+\infty}\frac{\left(x^2-\left(x^2+1\right)\right)\left(x^2+\sqrt{x^4-1}\right)}{\left(x^4-\left(x^4-1\right)\right)\left(x+\sqrt{x^2+1}\right)}=\lim_{x\mapsto+\infty}\frac{-\left(x^2+x^2\sqrt{1-\frac{1}{x^4}}\right)}{\left(x+x\sqrt{1+\frac{1}{x^2}}\right)}$$

$$\lim_{x\mapsto+\infty}\left(\frac{x-\sqrt{x^2+1}}{x^2-\sqrt{x^4-1}}\right)==\lim_{x\mapsto+\infty}\frac{-x\left(1+\sqrt{1-\frac{1}{x^4}}\right)}{1+\sqrt{1+\frac{1}{x^2}}}=-\infty$$

$$\lim_{x\mapsto-\infty}\left(\frac{x+1-\sqrt{1-x}}{x^2-\sqrt{x^2+2}}\right)=\lim_{x\mapsto-\infty}\frac{x\left(1+\frac{1}{x}-\frac{\sqrt{1-x}}{x}\right)}{x^2+x\sqrt{1+\frac{2}{x^2}}}=\lim_{x\mapsto-\infty}\frac{1+\frac{1}{x}+\sqrt{\frac{1-x}{x^2}}}{x\left(1+\frac{1}{x}\sqrt{1+\frac{2}{x^2}}\right)}=0$$

لدينا : $\forall x\in IR\ \left|\cos x+5\sin x^2\right|\le\left|\cos x\right|+\left|5\sin x^2\right|\le 1+5\le 6$

منه : $\forall x\in IR\ \left|\frac{\cos x+5\sin x^2}{x^4+x^2+1}\right|\le\frac{6}{x^4+x^2+1}$ ، وبما أن $\lim_{x\mapsto+\infty}\frac{1}{x^4+x^2+1}=0$ فإن:

$$\lim_{x\mapsto+\infty}\frac{\cos x+5\sin x^2}{x^4+x^2+1}=0$$

تمرين 2 :

$$\lim_{x\mapsto-1}\left(\frac{\sqrt{1-3x}-2}{x+1}\right)=\lim_{x\mapsto-1}\frac{1-3x-4}{(x+1)\left(\sqrt{1-3x}+2\right)}=\lim_{x\mapsto-1}\frac{-3}{\sqrt{1-3x}+2}=\frac{-3}{4}$$

$$\lim_{x\mapsto2^+}\frac{\sqrt{x^2-4}}{x-2}=\lim_{x\mapsto2^+}\sqrt{\frac{(x-2)(x+2)}{(x-2)^2}}=\lim_{x\mapsto2^+}\sqrt{\frac{x+2}{x-2}}=+\infty$$

$$\lim_{x\mapsto-2^-}\frac{\sqrt{x^2-4}}{x+2}=\lim_{x\mapsto2^-}-\frac{\sqrt{x^2-4}}{-(x+2)}=\lim_{x\mapsto2^+}-\sqrt{\frac{(x-2)(x+2)}{(x+2)^2}}=\lim_{x\mapsto2^+}-\sqrt{\frac{x-2}{x+2}}=-\infty$$

$$\lim_{x\mapsto1}\left(\frac{x^2+x-2}{x^3+4x^2-8x+3}\right)=\lim_{x\mapsto1}\frac{(x-1)(x+2)}{(x-1)\left(x^2+5x-3\right)}=1$$

$$\lim_{x\mapsto1}\left(\sin\left(\frac{5}{1-x^3}\right)\left(x^2-2x+1\right)\right)=\lim_{x\mapsto1}\sin\left(\frac{5}{1-x^3}\right)(x-1)^2=0$$

لأن : $\forall x\in IR\ \left|\sin\left(\frac{5}{1-x^3}\right)\right|\le 1\ \Rightarrow\ \forall x\in IR\ \left|\sin\left(\frac{5}{1-x^3}\right)(x-1)^2\right|\le(x-1)^2$

و $\lim_{x\mapsto1}(x-1)^2=0$ و ذلك حسب مصاديق تقارب نهاية دالة.

$\lim_{x\mapsto0^+}\frac{1}{x}+\cos\left(\frac{1}{x}\right)=+\infty$ لأن: $\forall x>0\ \frac{1}{x}+\cos\left(\frac{1}{x}\right)\ge\frac{1}{x}+1 \Leftarrow \forall x>0\ \cos\left(\frac{1}{x}\right)\ge+1$ و $\lim_{x\mapsto0^+}\frac{1}{x}+1=+\infty$

وذلك حسب مصاديق تقارب نهاية دالة.

تمرين 3 :

$$\lim_{x\mapsto 0}\left(\frac{1}{\sqrt{3}\cos x-\sin^2 x-\sqrt{3}}\right)=\lim_{x\mapsto 0}\left(\frac{1}{\sqrt{3}(\cos x-1)-\sin^2 x}\right)=\lim_{x\mapsto 0}\left(\frac{-1}{\sqrt{3}(1-\cos(x))+\sin^2 x}\right)$$

$$=\lim_{x\mapsto 0}\left(\frac{-\frac{1}{x^2}}{\sqrt{3}\left(\frac{1-\cos(x)}{x^2}\right)+\left(\frac{\sin x}{x}\right)^2}\right)=-\infty$$

(لأن: $\lim_{x\mapsto 0}\sqrt{3}\left(\frac{1-\cos(x)}{x^2}\right)+\left(\frac{\sin x}{x}\right)^2=\frac{\sqrt{3}}{2}+1$ و $\lim_{x\mapsto 0}-\frac{1}{x^2}=-\infty$)

$$\lim_{x\mapsto 0}\left(\frac{\sqrt{3}\cos x-\sin^2 x-\sqrt{3}}{x^2}\right)=\lim_{x\mapsto 0}\left(-\sqrt{3}\left(\frac{1-\cos(x)}{x^2}\right)-\left(\frac{\sin(x)}{x}\right)^2\right)=\frac{-\sqrt{3}}{2}-1$$

$$\lim_{x\mapsto \frac{\pi}{6}}\left(\frac{\cos x-\sqrt{3}\sin x}{6x-\pi}\right)=\lim_{x\mapsto \frac{\pi}{6}}\left(\frac{2\left(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x\right)}{6\left(x-\frac{\pi}{6}\right)}\right)=\lim_{x\mapsto \frac{\pi}{6}}\left(\frac{1}{3}\frac{\left(\sin\left(\frac{\pi}{6}\right)\cos x-\cos\left(\frac{\pi}{6}\right)\sin x\right)}{\left(x-\frac{\pi}{6}\right)}\right)$$

$$=\lim_{x\mapsto \frac{\pi}{6}}\left(\frac{1}{3}\frac{\sin\left(\frac{\pi}{6}-x\right)}{\left(x-\frac{\pi}{6}\right)}\right)==\lim_{x\mapsto \frac{\pi}{6}}\left(\frac{-1}{3}\frac{\sin\left(x-\frac{\pi}{6}\right)}{\left(x-\frac{\pi}{6}\right)}\right)=\lim_{t\mapsto 0}\left(\frac{-1}{3}\frac{\sin(t)}{t}\right)=\frac{-1}{3}$$

(قمنا بتغيير المتغير x وذلك بوضع $t=x-\frac{\pi}{6}$)، كما يمكن إجراء تغيير المتغير منذ البداية.

نضع $t=x-\frac{\pi}{4}$

$$\lim_{x\mapsto \frac{\pi}{4}}\left(\frac{\tan x-1}{2\cos x-\sqrt{2}}\right)=\lim_{x\mapsto \frac{\pi}{4}}\left(\frac{\tan x-\tan\left(\frac{\pi}{4}\right)}{2\cos x-\sqrt{2}}\right)=\lim_{x\mapsto \frac{\pi}{4}}\left(\frac{\left(1+\tan(x)\tan\left(\frac{\pi}{4}\right)\right)\tan\left(x-\frac{\pi}{4}\right)}{2\cos x-\sqrt{2}}\right)$$

$$\lim_{x\mapsto \frac{\pi}{4}}\left(\frac{\tan x-1}{2\cos x-\sqrt{2}}\right)=\lim_{t\mapsto 0}\left(\frac{\left(1+\tan\left(t+\frac{\pi}{4}\right)\right)\tan(t)}{2\cos\left(t+\frac{\pi}{4}\right)-\sqrt{2}}\right)=\lim_{t\mapsto 0}\left(\frac{\left(1+\tan\left(t+\frac{\pi}{4}\right)\right)\tan(t)}{2\left(\cos(t)\frac{\sqrt{2}}{2}-\sin(t)\frac{\sqrt{2}}{2}\right)-\sqrt{2}}\right)$$

$$=\lim_{t\mapsto 0}\left(\frac{\left(1+\tan\left(t+\frac{\pi}{4}\right)\right)\tan(t)}{-\sqrt{2}(1-\cos(t))-\sqrt{2}\sin(t)}\right)=\lim_{t\mapsto 0}\left(\frac{\left(1+\tan\left(t+\frac{\pi}{4}\right)\right)\frac{\tan(t)}{t}}{-\sqrt{2}\,t\frac{(1-\cos(t))}{t^2}-\sqrt{2}\frac{\sin(t)}{t}}\right)$$

$$=\frac{(1+1)\times 1}{-\sqrt{2}\times 0\times\frac{1}{2}-\sqrt{2}\times 1}=-\sqrt{2}$$

لاحظ أن استعمال الخاصية $\tan(a-b)=\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$

بالشكل $\tan(a)-\tan(b)=(1+\tan(a)\tan(b))\tan(a-b)$ أفضل من استعمال تغيير المتغير من البداية.

$$\lim_{x\mapsto\frac{\pi}{4}}\left(\frac{\sqrt{1-\cos(x)}-\sqrt{1-\sin(x)}}{1-\tan(x)}\right)=\lim_{x\mapsto\frac{\pi}{4}}\left(\frac{\sin(x)-\cos(x)}{(1+\tan(x))\tan\left(x-\frac{\pi}{4}\right)}\times\frac{1}{\sqrt{1-\cos(x)}+\sqrt{1-\sin(x)}}\right)$$

$$=\lim_{x\mapsto\frac{\pi}{4}}\left(\frac{\sqrt{2}\left(\sin(x)\cos\left(\frac{\pi}{4}\right)-\cos(x)\sin\left(\frac{\pi}{4}\right)\right)}{(1+\tan(x))\tan\left(x-\frac{\pi}{4}\right)}\times\frac{1}{\sqrt{1-\cos(x)}+\sqrt{1-\sin(x)}}\right)$$

$$=\lim_{x\mapsto\frac{\pi}{4}}\left(\frac{\sin\left(x-\frac{\pi}{4}\right)}{\tan\left(x-\frac{\pi}{4}\right)}\times\frac{\sqrt{2}}{(1+\tan(x))\left(\sqrt{1-\cos(x)}+\sqrt{1-\sin(x)}\right)}\right)$$

وبما أن :

$$\lim_{x\mapsto\frac{\pi}{4}}\frac{\sin\left(x-\frac{\pi}{4}\right)}{\tan\left(x-\frac{\pi}{4}\right)}=\lim_{t\mapsto 0}\frac{\sin(t)}{\tan(t)}=\lim_{t\mapsto 0}\frac{\frac{\sin(t)}{t}}{\frac{\tan(t)}{t}}=\frac{1}{1}=1$$

و

$$\lim_{x\mapsto\frac{\pi}{4}}\left(\frac{\sqrt{2}}{(1+\tan(x))\left(\sqrt{1-\cos(x)}+\sqrt{1-\sin(x)}\right)}\right)=\frac{\sqrt{2}}{(1+1)\left(\sqrt{1-\frac{\sqrt{2}}{2}}+\sqrt{1-\frac{\sqrt{2}}{2}}\right)}$$

$$=\frac{\sqrt{2}}{4\sqrt{\frac{2-\sqrt{2}}{2}}}=\frac{\sqrt{4+2\sqrt{2}}}{4}$$

فإن :

$$\lim_{x\mapsto\frac{\pi}{4}}\left(\frac{\sqrt{1-\cos(x)}-\sqrt{1-\sin(x)}}{1-\tan(x)}\right)=\frac{\sqrt{4+2\sqrt{2}}}{4}$$

تمرين 4 : $f_a(x)=\dfrac{1}{x+a}-\dfrac{a^2x^2}{x^3+a^3}$

1) لدينا: $x\neq -a \Leftrightarrow \begin{cases} x\neq -a \\ x^3\neq(-a)^3\end{cases} \Leftrightarrow \begin{cases} x+a\neq 0 \\ x^3+a^3\neq 0\end{cases} \Leftrightarrow x\in Df_a$ منه $Df_a=IR_{\setminus\{-a\}}$

2) لدينا : $f_a(x)=\dfrac{1}{x+a}-\dfrac{a^2x^2}{x^3+a^3}=\dfrac{x^2-ax+a^2}{x^3+a^3}-\dfrac{a^2x^2}{x^3+a^3}=\dfrac{(1-a^2)x^2-ax+a^2}{x^3+a^3}$

نضع : $g_a(x)=(1-a^2)x^2-ax+a^2$ ، إذن : $g_a(-a)=(1-a^2)a^2+a^2+a^2=3a^2-a^4=a^2(3-a^2)$

- إذا كان : $a=0$ فإن: $f_0(x)=\dfrac{x^2}{x^3}=\dfrac{1}{x}$ منه : $\lim\limits_{\substack{x\mapsto 0\\ x>0}} f_0(x)=+\infty$ و $\lim\limits_{\substack{x\mapsto 0\\ x<0}} f_0(x)=-\infty$

- إذا كان : $a^2<3$ و $a\neq 0$ (أي $a\in\left]-\sqrt{3};0\right[\cup\left]0;\sqrt{3}\right[$) فإن: $g_a(-a)>0$
 منه : $\lim\limits_{\substack{x\mapsto -a\\ x>-a}} f_a(x)=+\infty$ و $\lim\limits_{\substack{x\mapsto -a\\ x<-a}} f_a(x)=-\infty$

- إذا كان : $a^2>3$ و (أي $a\in\left]-\infty;-\sqrt{3}\right[\cup\left]\sqrt{3};+\infty\right[$) فإن: $g_a(-a)<0$
 منه : $\lim\limits_{\substack{x\mapsto -a\\ x>-a}} f_a(x)=-\infty$ و $\lim\limits_{\substack{x\mapsto -a\\ x<-a}} f_a(x)=+\infty$

- إذا كان : $a=\sqrt{3}$ فإن: $g_a(-\sqrt{3})=0$ ، منه :

$$\lim_{x\mapsto-\sqrt{3}} f_{\sqrt{3}}(x)=\lim_{x\mapsto-\sqrt{3}}\frac{-2x^2-\sqrt{3}x+3}{x^3+(\sqrt{3})^3}=\lim_{x\mapsto-\sqrt{3}}\frac{(x+\sqrt{3})(\sqrt{3}-2x)}{(x+\sqrt{3})(x^2-x\sqrt{3}+3)}=\lim_{x\mapsto-\sqrt{3}}\frac{\sqrt{3}-2x}{x^2-x\sqrt{3}+3}=\frac{3\sqrt{3}}{9}=\frac{\sqrt{3}}{3}$$

- إذا كان : $a=-\sqrt{3}$ فإن: $g_a(\sqrt{3})=0$ ، منه :

$$\lim_{x\mapsto\sqrt{3}} f_{\sqrt{3}}(x)=\lim_{x\mapsto\sqrt{3}}\frac{-2x^2+\sqrt{3}x+3}{x^3-(\sqrt{3})^3}=\lim_{x\mapsto\sqrt{3}}\frac{(x-\sqrt{3})(-\sqrt{3}-2x)}{(x-\sqrt{3})(x^2+x\sqrt{3}+3)}=\lim_{x\mapsto\sqrt{3}}\frac{-\sqrt{3}-2x}{x^2+x\sqrt{3}+3}=\frac{-3\sqrt{3}}{9}=\frac{-\sqrt{3}}{3}$$

خلاصة : القيم التي تجيب عن السؤال هي : $a=\sqrt{3}$ و $a=-\sqrt{3}$

استعملنا المحددة للتعميل

تمرين 5: $f(x)=\dfrac{mx^3+(m-2)x^2+(m-1)x+m-3}{x(x-2)(x-3)}$

1) لدينا: $x\in Df \Leftrightarrow x\neq 0$ et $x-2\neq 0$ et $x-3\neq 0$ ، إذن: $Df=IR\setminus\{0;2;3\}$

2) ندرس النهاية في اللانهاية:

- إذاكان $m=0$ فإن: $\lim\limits_{x\mapsto+\infty} f(x)=\lim\limits_{x\mapsto+\infty}\dfrac{-2x^2-x-3}{x(x-2)(x-3)}=\lim\limits_{x\mapsto+\infty}\dfrac{-2x^2}{x^3}=\lim\limits_{x\mapsto+\infty}\dfrac{-2}{x}=0$ و $\lim\limits_{x\mapsto-\infty} f(x)=0$
- إذاكان $m\neq 0$ فإن: $\lim\limits_{x\mapsto+\infty} f(x)=\lim\limits_{x\mapsto+\infty}\dfrac{mx^3}{x^3}=m$ و $\lim\limits_{x\mapsto-\infty} f(x)=m$

يمكن أن نلخص الحالات في النتيجة التالية، $\lim\limits_{x\mapsto-\infty} f(x)=\lim\limits_{x\mapsto+\infty} f(x)=m \quad \forall m\in IR$

نضع: $g(x)=mx^3+(m-2)x^2+(m-1)x+m-3$

$g(0)=m-3$ و $g(2)=8m+4(m-2)+2(m-1)+m-3=15m-13$

و $g(3)=27m+9(m-2)+3(m-1)+m-3=40m-24$

ندرس النهاية في 0 :

- إذاكان $m=3$ فإن: $\lim\limits_{x\mapsto 0} f(x)=\lim\limits_{x\mapsto 0}\dfrac{3x^3+x^2+2x}{x(x-2)(x-3)}=\lim\limits_{x\mapsto 0}\dfrac{3x^2+x+2}{(x-2)(x-3)}=\dfrac{2}{6}=\dfrac{1}{3}$
- إذاكان $m>3$ فإن: $\lim\limits_{\substack{x\mapsto 0\\ x>0}} f(x)=+\infty \left(\dfrac{\ell>0}{0^+}\right)$ و $\lim\limits_{\substack{x\mapsto 0\\ x<0}} f(x)=-\infty \left(\dfrac{\ell>0}{0^-}\right)$
- إذاكان $m<3$ فإن: $\lim\limits_{\substack{x\mapsto 0\\ x>0}} f(x)=-\infty \left(\dfrac{\ell<0}{0^+}\right)$ و $\lim\limits_{\substack{x\mapsto 0\\ x<0}} f(x)=+\infty \left(\dfrac{\ell<0}{0^-}\right)$

تمرين 6: $f(x)=\dfrac{x^2\sqrt{x+2}-8}{4-x^2}$

1) لدينا: $x\in Df \Leftrightarrow \begin{cases}4-x^2\neq 0\\ x+2\geq 0\end{cases} \Leftrightarrow \begin{cases}x\neq 2 \text{ et } x\neq -2\\ x\geq -2\end{cases} \Leftrightarrow \begin{cases}x\neq 2\\ x>-2\end{cases} \Leftrightarrow x\in]-2;2[\cup]2;+\infty[$

إذن: $Df=]-2;2[\cup]2;+\infty[$

2) إذاكان $-\infty$: $\lim\limits_{x\mapsto+\infty} f(x)=\lim\limits_{x\mapsto+\infty}\dfrac{x^2\sqrt{x+2}-8}{4-x^2}=\lim\limits_{x\mapsto+\infty}\dfrac{x^2\left(\sqrt{x+2}-\dfrac{8}{x^2}\right)}{x^2\left(\dfrac{4}{x^2}-1\right)}=\lim\limits_{x\mapsto+\infty}\dfrac{\sqrt{x+2}-\dfrac{8}{x^2}}{\dfrac{4}{x^2}-1}=-\infty$

3) $\lim\limits_{x\mapsto 2} f(x)=\lim\limits_{x\mapsto 2}\dfrac{x^2\sqrt{x+2}-8}{4-x^2}=\lim\limits_{x\mapsto 2}\dfrac{x^2\left(\sqrt{x+2}-2\right)+2x^2-8}{4-x^2}=\lim\limits_{x\mapsto 2}\dfrac{x^2\dfrac{x+2-4}{\sqrt{x+2}+2}+2(x^2-4)}{4-x^2}$

$=\lim\limits_{x\mapsto 2}\dfrac{x^2\dfrac{x-2}{\sqrt{x+2}+2}}{4-x^2}-2=\lim\limits_{x\mapsto 2}\dfrac{-x^2}{(x+2)\left(\sqrt{x+2}+2\right)}-2=\dfrac{-4}{16}-2=\dfrac{-9}{4}$

إذن f تقبل تمديدا بالاتصال في 2 تمديده هو: $\begin{cases}f(x)=\dfrac{x^2\sqrt{x+2}-8}{4-x^2};\ x\in]-2;2[\cup]2;+\infty[\\ f(2)=\dfrac{9}{4}\end{cases}$