السنة 2 بكالوريا علوم رياضية	الحساب التكاملي حلول مقترحة	سلسلة 1

تمرين 1 :

$$\int_0^1 x\sqrt{x}\,dx = \int_0^1 x^{\frac{3}{2}}\,dx = \left[\frac{2}{5}x^{\frac{5}{2}}\right]_0^1 = \frac{2}{5}$$

$$\int_0^{\ln(2)} e^{2x} + \frac{1}{e^{3x}} + \sqrt{e^x}\,dx = \int_0^{\ln(2)} e^{2x} + e^{-3x} + e^{\frac{x}{2}}\,dx = \left[\frac{1}{2}e^{2x} + \frac{-1}{3}e^{-3x} + 2e^{\frac{x}{2}}\right]_0^{\ln 2}$$

$$= \left(2 - \frac{1}{3}\times\frac{1}{8} + 2\sqrt{2}\right) - \left(\frac{1}{2} - \frac{1}{3} + 2\right) = \frac{-1}{24} + 2\sqrt{2} - \frac{1}{6} = 2\sqrt{2} - \frac{5}{24}$$

$$\int_1^2 \frac{1+x}{x^2}\,dx = \int_1^2 \frac{1}{x^2} + \frac{1}{x}\,dx = \left[\frac{-1}{x} + \ln(x)\right]_1^2 = \left(\frac{-1}{2} + \ln(2)\right) - (-1) = \frac{1}{2} + \ln(2)$$

$$\int_1^2 \frac{x^2}{1+x}\,dx = \int_1^2 \frac{x^2 - 1 + 1}{1+x}\,dx = \int_1^2 x - 1 - \frac{1}{1+x}\,dx = \left[\frac{1}{2}x^2 - x - \ln(x+1)\right]_1^2 = (2 - 2 - \ln(3)) - \left(\frac{1}{2} - 1 - \ln(2)\right)$$

$$= \ln\left(\frac{2}{3}\right) + \frac{1}{2}$$

$$\int_0^{\frac{\pi}{4}} \tan x\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx = -\int_0^{\frac{\pi}{4}} \frac{-\sin x}{\cos x}\,dx = -\left[\ln(|\cos x|)\right]_0^{\frac{\pi}{4}} = -\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{\ln(2)}{2}$$

$$\int_0^{\frac{\pi}{4}} \tan^2 x\,dx = \int_0^{\frac{\pi}{4}} \tan^2 x + 1 - 1\,dx = \left[\tan x - x\right]_0^{\frac{\pi}{4}} = 1 - \frac{\pi}{4}$$

$$\int_2^3 \frac{2}{x^2 - 1}\,dx = \int_2^3 \frac{(x+1) - (x-1)}{x^2 - 1}\,dx = \int_2^3 \frac{1}{x-1} - \frac{1}{x+1}\,dx = \left[\ln(|x-1|) - \ln(|x+1|)\right]_2^3$$

$$= (\ln(2) - \ln(4)) - (\ln(1) - \ln(3)) = \ln\left(\frac{3}{2}\right)$$

$$\int_1^e \frac{\ln(x)}{x}\,dx = \left[\frac{1}{2}\ln^2(x)\right]_1^e = \frac{1}{2}$$

$$\int_0^2 \frac{1}{|x-1| + 1}\,dx = \int_0^1 \frac{1}{1 - x + 1}\,dx + \int_1^2 \frac{1}{x - 1 + 1}\,dx = \int_0^1 \frac{-1}{x-2}\,dx + \int_1^2 \frac{1}{x}\,dx = \left[-\ln(|x-2|)\right]_0^1 + \left[\ln(|x|)\right]_1^2$$

$$= \ln 2 + \ln 2 = 2\ln 2$$

$$\int_0^{\ln(2)} \frac{1}{e^x + 1}\,dx = \int_0^{\ln(2)} \frac{1 + e^x - e^x}{e^x + 1}\,dx = \int_0^{\ln(2)} 1 - \frac{e^x}{e^x + 1}\,dx = \left[x - \ln(e^x + 1)\right]_0^{\ln(2)} = \ln(2) - \ln(3) + \ln(2) = \ln\left(\frac{4}{3}\right)$$

$$\int_e^{e^2} \frac{1}{x\ln(x)}\,dx = \int_e^{e^2} \frac{\frac{1}{x}}{\ln(x)}\,dx = \left[\ln(\ln x)\right]_e^{e^2} = \ln(2)$$

$$\int_0^{\frac{\pi}{2}} \sin^5(x)\,dx = \int_0^{\frac{\pi}{2}} (\sin^2(x))^2 \sin(x)\,dx = \int_0^{\frac{\pi}{2}} (1 - \cos^2(x))^2 \sin(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x - 2\cos^2 x \sin x + \cos^4 x \sin x\,dx$$

$$\int_0^{\frac{\pi}{2}} \sin^5(x)\,dx = \left[-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x\right]_0^{\frac{\pi}{2}} = -\left(-1 + \frac{2}{3} - \frac{1}{5}\right) = \frac{8}{15}$$

$$\int_0^1 \frac{x^3}{x^2+1}dx = \int_0^1 \frac{x^3+x-x}{x^2+1}dx = \int_0^1 x - \frac{x}{x^2+1}dx = \left[\frac{1}{2}x^2 - \frac{1}{2}\ln\left(x^2+1\right)\right]_0^1 = \frac{1}{2} - \frac{\ln(2)}{2}$$

$$\int_2^3 \frac{1}{x^4-1}dx = \frac{1}{2}\int_2^3 \frac{\left(x^2+1\right)-\left(x^2-1\right)}{\left(x^2-1\right)\left(x^2+1\right)}dx = \frac{1}{2}\int_2^3 \frac{1}{x^2-1} - \frac{1}{x^2+1}dx = \frac{1}{2}\int_2^3 \frac{1}{2}\frac{(x+1)-(x-1)}{(x-1)(x+1)} - \frac{1}{x^2+1}dx$$

$$= \frac{1}{2}\int_2^3 \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\frac{1}{x+1} - \frac{1}{x^2+1}dx = \frac{1}{2}\left[\frac{1}{2}\ln(|x-1|) - \frac{1}{2}\ln(|x+1|) - \text{Arctan } x\right]_2^3$$

$$= \frac{1}{2}\left[\frac{1}{2}\ln\left(\frac{|x-1|}{|x+1|}\right) - \text{Arctan } x\right]_2^3 = \frac{1}{2}\left(\frac{1}{2}\ln\left(\frac{1}{2}\right) - \text{Arctan}\,3 + \text{Arctan}\,2\right)$$

$$\int_2^3 \frac{1}{x^4-1}dx = \frac{-\ln(2)}{4} + \frac{\text{Arctan } 2 - \text{Arctan } 3}{2}$$

$$\int_0^1 \frac{\text{Arctan } x}{x^2+1}dx = \int_0^1 \text{Arctan } x \times \frac{1}{x^2+1}dx = \left[\frac{1}{2}\text{Arctan}^2\, x\right]_0^1 = \frac{\text{®}^2}{32}$$

$$\int_1^2 \frac{1}{\sqrt{x}\,(x+1)}dx = \int_1^2 2\frac{\frac{1}{2\sqrt{x}}}{\left(1+\left(\sqrt{x}\right)^2\right)}dx = \left[2\text{ Arctan}\sqrt{x}\right]_1^2 = 2\,\text{Arctan}\sqrt{2} - \frac{\text{®}}{2}$$

تمرين 2 :

نضع : $t = x+2$ منه : $x = t-2$ و $dx = dt$ و $x=0 \Rightarrow t=2$ ، $x=1 \Rightarrow t=3$ منه :

$$\int_0^1 \frac{x}{\sqrt{x+2}}dx = \int_2^3 \frac{t-2}{\sqrt{t}}dt = \int_2^3 \frac{t}{\sqrt{t}} - \frac{2}{\sqrt{t}}dt = \int_2^3 t^{\frac{1}{2}} - 2t^{\frac{-1}{2}}dt = \left[\frac{2}{3}t^{\frac{3}{2}} - 4t^{\frac{1}{2}}\right]_2^3$$

$$\int_0^1 \frac{x}{\sqrt{x+2}}dx = \frac{2}{3}\times 3\sqrt{3} - 4\sqrt{3} - \frac{2}{3}\times 2\sqrt{2} + 2\sqrt{2} = -2\sqrt{3} + \frac{2}{3}\sqrt{2}$$

نضع : $t = 1-x$ منه : $x = 1-t$ و $dx = -dt$ و $x=0 \Rightarrow t=1$ ، $x=1 \Rightarrow t=0$ منه :

$$\int_0^1 x(1-x)^{2015}dx = \int_1^0 -(1-t)t^{2015}dt = \int_0^1 (1-t)t^{2015}dt = \int_0^1 t^{2015} - t^{2016}dt = \left[\frac{t^{2015}}{2015} - \frac{t^{2016}}{2016}\right]_0^1$$

$$\int_0^1 x(1-x)^{2015}dx = \frac{1}{2015\times 2016} = \frac{1}{4062240}$$

نضع : $t = \frac{2x+1}{\sqrt{3}}$ منه : $x = \frac{t\sqrt{3}-1}{2}$ و $dx = \frac{\sqrt{3}}{2}dt$ و $x=0 \Rightarrow t=\frac{1}{\sqrt{3}}$ ، $x=1 \Rightarrow t=\sqrt{3}$ منه :

$$\int_0^1 \frac{1}{x^2+x+1}dx = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{\frac{3t^2-2\sqrt{3}+1}{4} + \frac{t\sqrt{3}-1}{2} + 1}\frac{\sqrt{3}}{2}dt = \frac{\sqrt{3}}{2}\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{4}{3t^2-2\sqrt{3}t+1+2\sqrt{3}t-2+4}dt$$

$$\int_0^1 \frac{1}{x^2+x+1}dx = \frac{\sqrt{3}}{2}\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{4}{3t^2+3}dt = \frac{2\sqrt{3}}{3}\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{t^2+1}dt = \frac{2\sqrt{3}}{3}\left[\text{Arctan t}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} = \frac{2\sqrt{3}}{3}\left(\frac{\text{®}}{3} - \frac{\text{®}}{6}\right) = \frac{\sqrt{3}\,\text{®}}{9}$$

لا يصح أن نكتب : $dx = \frac{\sqrt{3}\,dt}{2}$ مثل المتجهات لا يصح أن نكتب مثلا $\overrightarrow{AB} = \frac{\sqrt{3}\,\overrightarrow{AC}}{2}$

نضع : $t = \sqrt{e^x+1}$ منه : $x = \ln\left(t^2-1\right)$ و $dx = \frac{2t}{t^2-1}dt$ و $x=0 \Rightarrow t=\sqrt{2}$ ، $x=1 \Rightarrow t=\sqrt{e+1}$ منه :

$$\int_0^1 \frac{e^{2x}}{\sqrt{e^x+1}}dx = \int_{\sqrt{2}}^{\sqrt{e+1}} \frac{(t^2-1)^2}{t}\frac{2t}{t^2-1}dt = 2\int_{\sqrt{2}}^{\sqrt{e+1}} t^2-1\,dt = 2\left[\frac{t^3}{3}-t\right]_{\sqrt{2}}^{\sqrt{e+1}}$$

$$\int_0^1 \frac{e^{2x}}{\sqrt{e^x+1}}dx = 2\left(\frac{(e+1)\sqrt{e+1}}{3}-\sqrt{e+1}-\frac{2\sqrt{2}}{3}+\sqrt{2}\right)=\frac{2}{3}\left((e-2)\sqrt{e+1}+\sqrt{2}\right)$$

نضع : $t=\sqrt{\sqrt{x}+1}$ منه : $x=(t^2-1)^2$ و $dx=4t(t^2-1)dt$ و $x=0 \Rightarrow t=1$ ، $x=1 \Rightarrow t=\sqrt{2}$: منه

$$\int_0^1\sqrt{\sqrt{x}+1}\,dx = \int_1^{\sqrt{2}}4t^2(t^2-1)dx = 4\int_1^{\sqrt{2}} t^4-t^2\,dx = 4\left[\frac{t^5}{5}-\frac{t^3}{3}\right]_1^{\sqrt{2}} = 4\left(\frac{4\sqrt{2}}{5}-\frac{2\sqrt{2}}{3}-\frac{1}{5}+\frac{1}{3}\right)=\frac{8}{15}(\sqrt{2}+1)$$

نعلم أن الدالة $x\to\sin x$ متصلة و تزايدية قطعا على $\left[0;\frac{®}{2}\right]$ ، إذن فهي تقبل دالة عكسية $¨(x)$ معرفة من $[0;1]$ نحو $\left[0;\frac{®}{2}\right]$ ، نضع : $t=¨(x)$ منه : $x=\sin t$ منه $dx=\cos t\,dt$ و

$\sqrt{1-x^2}=\sqrt{1-\sin^2 t}=\sqrt{\cos^2 t}=\cos t$ و $x=0 \Rightarrow t=0$ ، $x=1 \Rightarrow t=\frac{®}{2}$: منه

$$\int_0^1\sqrt{1-x^2}\,dx=\int_0^{\frac{®}{2}}\cos^2 t\,dt=\int_0^{\frac{®}{2}}\frac{1+\cos 2t}{2}dt=\frac{1}{2}\left[t+\frac{\sin(2t)}{2}\right]_0^{\frac{®}{2}}=\frac{®}{4}$$

لا يصح أن نضع $x=\sin t$ مباشرة (رغم أنها إشارة في السؤال) ، لأن الوضع يتم عن طريق دالة للمتغير x ، فطريقة تغيير المتغير مجرد اختصار لخاصية تكامل دالة باستعمال مركب دالتين، و التي تخضع لشروط يجب مراعاتها

تمرين 3 :

$$\int_0^1(3+2x)e^x\,dx=\int_0^1(3+2x)(e^x)'\,dx=\left[(3+2x)e^x\right]_0^1-\int_0^1(3+2x)'\,e^x\,dx=\left[(3+2x)e^x\right]_0^1-\int_0^1 2e^x\,dx$$

$$\int_0^1(3+2x)e^x\,dx=\left[(3+2x)e^x-2e^x\right]_0^1=\left[(1+2x)e^x\right]_0^1=3e-1$$

$$\int_1^2\ln(x)\,dx=\int_1^2 x'\ln(x)\,dx=\left[x\ln(x)\right]_1^2-\int_1^2 x\,(\ln(x))'\,dx=\left[x\ln(x)\right]_1^2-\int_1^2 1dx=\left[x\ln(x)-x\right]_1^2$$

$$\int_1^2\ln(x)\,dx=2\ln 2-2+1=2\ln 2-1$$

$$\int_0^{\frac{®}{2}}x\cos(x)dx=\int_0^{\frac{®}{2}}x\,(\sin(x))'\,dx=\left[x\sin(x)\right]_0^{\frac{®}{2}}-\int_0^{\frac{®}{2}}x'\sin(x)dx=\left[x\sin(x)\right]_0^{\frac{®}{2}}-\int_0^{\frac{®}{2}}\sin(x)dx$$

$$\int_0^{\frac{®}{2}}x\cos(x)dx=\left[x\sin(x)+\cos(x)\right]_0^{\frac{®}{2}}=\frac{®}{2}-1$$

$$\int_1^2 x\ln(x)\,dx=\int_1^2\left(\frac{x^2}{2}\right)'\ln(x)\,dx=\left[\frac{x^2}{2}\ln(x)\right]_1^2-\int_1^2\frac{x^2}{2}(\ln(x))'\,dx=\left[\frac{x^2}{2}\ln(x)\right]_1^2-\int_1^2\frac{x}{2}\,dx$$

$$\int_1^2 x\ln(x)\,dx=\left[\frac{x^2}{2}\ln(x)-\frac{x^2}{4}\right]_1^2=2\ln 2-1+\frac{1}{4}=2\ln 2-\frac{3}{4}$$

$$\int_1^e\frac{\ln(x)}{x^2}dx=\int_1^e\left(-\frac{1}{x}\right)'\ln(x)dx=\left[\frac{-\ln(x)}{x}\right]_1^e-\int_1^e\frac{-1}{x}(\ln(x))'\,dx=\left[\frac{-\ln(x)}{x}\right]_1^e+\int_1^e\frac{1}{x^2}dx$$

$$\int_1^e\frac{\ln(x)}{x^2}dx=\left[\frac{-\ln(x)}{x}-\frac{1}{x}\right]_1^e=\frac{-1}{e}-\frac{1}{e}+1=\frac{e-2}{e}$$

$$\int_1^2 x\arctan x\,dx=\int_1^2\left(\frac{x^2}{2}\right)'\arctan x\,dx=\left[\frac{x^2}{2}\arctan x\right]_1^2-\int_1^2\frac{x^2}{2}(\arctan x)'\,dx$$

$$=\left[\frac{x^2}{2}\arctan x\right]_1^2-\frac{1}{2}\int_1^2\frac{x^2}{x^2+1}\,dx=\left[\frac{x^2}{2}\arctan x\right]_1^2-\frac{1}{2}\int_1^2 1-\frac{1}{x^2+1}\,dx$$

$$=\left[\frac{x^2}{2}\arctan x-\frac{1}{2}(x-\arctan x)\right]_1^2=\frac{1}{2}\left[(x^2+1)\arctan x-x\right]_1^2=\frac{1}{2}\left(5\arctan 2-2-\frac{\text{®}}{2}+1\right)$$

$$\int_1^2 x\arctan x\,dx=\frac{10\arctan 2-2-\text{®}}{4}$$

$$\int_0^{\frac{\text{®}}{2}}\sin x\,e^x\,dx=\int_0^{\frac{\text{®}}{2}}\sin x\,(e^x)'\,dx=\left[\sin x\,e^x\right]_0^{\frac{\text{®}}{2}}-\int_0^{\frac{\text{®}}{2}}(\sin x)'\,e^x\,dx=\left[\sin x\,e^x\right]_0^{\frac{\text{®}}{2}}-\int_0^{\frac{\text{®}}{2}}\cos x\,e^x\,dx$$

$$=\left[\sin x\,e^x\right]_0^{\frac{\text{®}}{2}}-\int_0^{\frac{\text{®}}{2}}\cos x\,(e^x)'\,dx=\left[\sin x\,e^x\right]_0^{\frac{\text{®}}{2}}-\left(\left[\cos x\,e^x\right]_0^{\frac{\text{®}}{2}}-\int_0^{\frac{\text{®}}{2}}(\cos x)'\,e^x\,dx\right)$$

$$\int_0^{\frac{\text{®}}{2}}\sin x\,e^x\,dx=\left[\sin x\,e^x\right]_0^{\frac{\text{®}}{2}}-\left(\left[\cos x\,e^x\right]_0^{\frac{\text{®}}{2}}+\int_0^{\frac{\text{®}}{2}}\sin x\,e^x\,dx\right)=\left[\sin x\,e^x-\cos x\,e^x\right]_0^{\frac{\text{®}}{2}}-\int_0^{\frac{\text{®}}{2}}\sin x\,e^x\,dx$$

منه: $2\int_0^{\frac{\text{®}}{2}}\sin x\,e^x\,dx=\left[(\sin x-\cos x)e^x\right]_0^{\frac{\text{®}}{2}}$ منه: $\int_0^{\frac{\text{®}}{2}}\sin x\,e^x\,dx=\frac{1}{2}\left(e^{\frac{\text{®}}{2}}+1\right)$

في هذا التكامل الخاص قمنا بالتعبير عن هذا التكامل بدلالة نفسه وبذلك تمكنا من تحديد قيمته

$$\int_1^e(\ln x)^2\,dx=\int_1^e x'(\ln x)^2\,dx=\left[x(\ln x)^2\right]_1^e-\int_1^e x\left((\ln x)^2\right)'\,dx=\left[x(\ln x)^2\right]_1^e-\int_1^e 2\ln x\,dx$$

$$=\left[x(\ln x)^2\right]_1^e-2\int_1^e x'\ln x\,dx=\left[x(\ln x)^2\right]_1^e-2\left(\left[x\ln x\right]_1^e-\int_1^e x(\ln x)'\,dx\right)$$

$$\int_1^e(\ln x)^2\,dx=\left[x(\ln x)^2-2x\ln x\right]_1^e+2\int_1^e 1\,dx=\left[x(\ln x)^2-2x\ln x+2x\right]_1^e=e-2e+2e-2=e-2$$

$$\int_0^1\frac{x+1}{e^x}\,dx=\int_0^1(x+1)e^{-x}\,dx=\int_0^1(x+1)\left(-e^{-x}\right)'\,dx=\left[-(x+1)e^{-x}\right]_0^1-\int_0^1(x+1)'\left(-e^{-x}\right)dx$$

$$\int_0^1\frac{x+1}{e^x}\,dx=\left[-(x+1)e^{-x}\right]_0^1+\int_0^1 e^{-x}\,dx=\left[-(x+1)e^{-x}-e^{-x}\right]_0^1=\left[\frac{-x-2}{e^x}\right]_0^1=\frac{-3}{e}+2=\frac{2e-3}{e}$$

تمرين 4 : $\forall x\in IR_{-\{-1,3\}}\quad \frac{-3x^2+7x+2}{x^2-2x-3}=a+\frac{b}{x+1}+\frac{c}{x-3}$

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لدينا : $a+\frac{b}{x+1}+\frac{c}{x-3}=a+\frac{bx-3b+cx+c}{(x+1)(x-3)}==\frac{ax^2-2ax-3a+bx-3b+cx+c}{x^2-2x-3}$

منه : $\forall x\in IR_{-\{-1,3\}}\quad ax^2+(-2a+b+c)x+(-3a-3b+c)=-3x^2+7x+2$

منه : $\begin{cases}a=-3\\-2a+b+c=7\\-3a-3b+c=2\end{cases}$ منه : $\begin{cases}a=-3\\b+c=1\\-3b+c=-7\end{cases}$ منه : $\begin{cases}a=-3\\c=1-b\\4b=8\end{cases}$ منه : $\begin{cases}a=-3\\c=-1\\b=2\end{cases}$

هناك طرق أخرى للجواب عن هذا السؤال كالقسمة الإقليدية

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$$I=\int_0^2\frac{-3x^2+7x+2}{x^2-2x-3}\,dx=\int_0^2-3+\frac{2}{x+1}-\frac{1}{x-3}\,dx=\left[-3x+2\ln(|x+1|)-\ln(|x-3|)\right]_0^2$$

$$I=-6+2\ln(3)+\ln(3)=3\ln(3)-6$$

تمرين 5 :

	$\forall x \in IR \quad \dfrac{e^{2x}-1}{e^{2x}+1}=\dfrac{e^{x}\left(e^{x}-e^{-x}\right)}{e^{x}\left(e^{x}+e^{-x}\right)}=\dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$	1
	$I=\int_{0}^{1}\dfrac{e^{2t}-1}{e^{2t}+1}\,dt==\int_{0}^{1}\dfrac{e^{t}-e^{-t}}{e^{t}+e^{-t}}dt=\left[\ln\left(e^{t}+e^{-t}\right)\right]_{0}^{1}=\ln\left(\dfrac{e^{2}+1}{2e}\right)$	2