السنة 1 بكالوريا علوم رياضية	الحساب المثلثـــــي حلول مقترحة	سلسلة 3

تمرين 1 :

$\sin(2x)+\sin(-5x)=0 \Leftrightarrow \sin(2x)=\sin(5x)$

لدينا : $\sin(2x)+\sin(-5x)=0 \Leftrightarrow 2x=5x+2k\pi \,/\, k\in Z \quad ou \quad 2x=\pi-5x+2k\pi \,/\, k\in Z$

$\sin(2x)+\sin(-5x)=0 \Leftrightarrow -3x=2k\pi \,/\, k\in Z \quad ou \quad 7x=(2k+1)\pi \,/\, k\in Z$

بالتالي : $S=\left\{-\frac{2k}{3}\pi \,/\, k\in Z\right\}\cup\left\{\frac{(2k+1)\pi}{7} \,/\, k\in Z\right\}$ أو أيضا : $S=\left\{\frac{2k}{3}\pi \,/\, k\in Z\right\}\cup\left\{\frac{(2k+1)\pi}{7} \,/\, k\in Z\right\}$

يمكن أحيانا تبسيط تعبير مجموعة الحلول (أعلاه : $-k\in Z \Leftrightarrow k\in Z$)

$\sin 3x-\cos x=0 \Leftrightarrow \sin 3x=\cos x \Leftrightarrow \cos\left(\frac{\pi}{2}-3x\right)=\cos(x)$

$\sin 3x-\cos x=0 \Leftrightarrow \frac{\pi}{2}-3x=x+2k\pi \,/\, k\in Z \quad ou \quad \frac{\pi}{2}-3x=-x+2k\pi \,/\, k\in Z$

$\sin 3x-\cos x=0 \Leftrightarrow 4x=\frac{\pi}{2}-2k\pi \,/\, k\in Z \quad ou \quad 2x=\frac{\pi}{2}-2k\pi \,/\, k\in Z$

$\sin 3x-\cos x=0 \Leftrightarrow x=\frac{\pi}{8}-\frac{k\pi}{2} \,/\, k\in Z \quad ou \quad x=\frac{\pi}{4}-k\pi \,/\, k\in Z$

بالتالي : $S=\left\{\frac{\pi}{8}-\frac{k\pi}{2} \,/\, k\in Z\right\}\cup\left\{\frac{\pi}{4}-k\pi \,/\, k\in Z\right\}$ أو أيضا : $S=\left\{\frac{\pi}{8}+\frac{k\pi}{2} \,/\, k\in Z\right\}\cup\left\{\frac{\pi}{4}+k\pi \,/\, k\in Z\right\}$

مجموعة صلاحية المعادلة هي : $D=IR-\left\{\frac{\pi}{2}+k\pi \,/\, k\in Z\right\}$

لدينا في هذه المجموعة : $\sqrt{3}\tan x+1=0 \Leftrightarrow \tan x=\tan\left(\frac{-\pi}{6}\right) \Leftrightarrow x=\frac{-\pi}{6}+k\pi \,/\, k\in Z$

بالتالي : $S=\left\{\frac{-\pi}{6}+k\pi \,/\, k\in Z\right\}\cap D=\left\{\frac{-\pi}{6}+k\pi \,/\, k\in Z\right\}$

$\sin 2x-2\cos^2 x=0 \Leftrightarrow 2\sin x\cos x-2\cos^2 x=0$

$\sin 2x-2\cos^2 x=0 \Leftrightarrow 2\cos x(\sin x-\cos x)=0$

$\sin 2x-2\cos^2 x=0 \Leftrightarrow \cos x=0 \quad ou \quad \cos(x)=\sin(x)$

$\sin 2x-2\cos^2 x=0 \Leftrightarrow \cos x=0 \quad ou \quad \cos(x)=\cos\left(\frac{\pi}{2}-x\right)$

$\sin 2x-2\cos^2 x=0 \Leftrightarrow \left(x=\frac{\pi}{2}+k\pi \,/\, k\in Z\right) ou \left(\frac{\pi}{2}-x=x+2k\pi \,/\, k\in Z\right) ou \left(\frac{\pi}{2}-x=-x+2k\pi \,/\, k\in Z\right)$

$\sin 2x-2\cos^2 x=0 \Leftrightarrow \left(x=\frac{\pi}{2}+k\pi \,/\, k\in Z\right) ou \left(2x=\frac{\pi}{2}-2k\pi \,/\, k\in Z\right) ou \left(\frac{\pi}{2}=2k\pi \,/\, k\in Z\right)$

$\sin 2x-2\cos^2 x=0 \Leftrightarrow \left(x=\frac{\pi}{2}+k\pi \,/\, k\in Z\right) ou \left(x=\frac{\pi}{4}-k\pi \,/\, k\in Z\right) ou \left(k=\frac{1}{4} \,/\, k\in Z\right)$

$\sin 2x-2\cos^2 x=0 \Leftrightarrow \left(x=\frac{\pi}{2}+k\pi \,/\, k\in Z\right) ou \left(x=\frac{\pi}{4}+k\pi \,/\, k\in Z\right)$

بالتالي : $S=\left\{\frac{\pi}{2}+k\pi \,/\, k\in Z\right\}\cup\left\{\frac{\pi}{4}+k\pi \,/\, k\in Z\right\}$

نذكر بالقواعد:

$\cos a = \cos b \Leftrightarrow a = b + 2k® \quad ou \quad a = -b + 2k® \quad / k \in Z$

$\cos a = 1 \Leftrightarrow a = 2k® \; / k \in Z \quad ; \quad \cos a = -1 \Leftrightarrow a = ® + 2k® \; / k \in Z \; ; \; \cos a = 0 \Leftrightarrow a = \frac{®}{2} + k® \; / k \in Z$

$\sin a = \sin b \Leftrightarrow a = b + 2k® \; / k \in Z \quad ou \quad a = ® - b + 2k® \; / k \in Z$

$\sin a = 1 \Leftrightarrow a = \frac{®}{2} + 2k® \; / k \in Z \quad ; \quad \sin a = -1 \Leftrightarrow a = \frac{-®}{2} + 2k® \; / k \in Z \; ; \; \sin a = 0 \Leftrightarrow a = k® \; / k \in Z$

$\tan a = \tan b \Leftrightarrow a = b + k® \; / k \in Z$

تمرين 2 :

لدينا :

$$\sin(x) + \sqrt{3}\cos x = \sqrt{3} \Leftrightarrow \frac{1}{2}\sin(x) + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{3}}{2}$$

$$\sin(x) + \sqrt{3}\cos x = \sqrt{3} \Leftrightarrow \sin\left(\frac{®}{6}\right)\sin(x) + \cos\left(\frac{®}{6}\right)\cos x = \frac{\sqrt{3}}{2}$$

$$\sin(x) + \sqrt{3}\cos x = \sqrt{3} \Leftrightarrow \cos\left(x - \frac{®}{6}\right) = \cos\left(\frac{®}{6}\right)$$

$$\sin(x) + \sqrt{3}\cos x = \sqrt{3} \Leftrightarrow x - \frac{®}{6} = \frac{®}{6} + 2k® \; / k \in Z \quad ou \quad x - \frac{®}{6} = \frac{-®}{6} + 2k® \; / k \in Z$$

$$\sin(x) + \sqrt{3}\cos x = \sqrt{3} \Leftrightarrow x = \frac{®}{3} + 2k® \; / k \in Z \quad ou \quad x = 2k® \; / k \in Z$$

بالتالي : $S = \left\{\frac{®}{3} + 2k® \; / k \in Z\right\} \cup \{2k® \; / k \in Z\}$

لدينا :

$$\sin x - \cos x = -1 \Leftrightarrow \frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x = -\frac{1}{\sqrt{2}}$$

$$\sin x - \cos x = -1 \Leftrightarrow \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$$

$$\sin x - \cos x = -1 \Leftrightarrow \cos\left(\frac{®}{4}\right)\cos x - \cos\left(\frac{®}{4}\right)\sin x = \cos\left(\frac{®}{4}\right)$$

$$\sin x - \cos x = -1 \Leftrightarrow \cos\left(x + \frac{®}{4}\right) = \cos\left(\frac{®}{4}\right)$$

$$\sin x - \cos x = -1 \Leftrightarrow x + \frac{®}{4} = \frac{®}{4} + 2k® \; / k \in Z \quad ou \quad x + \frac{®}{4} = \frac{-®}{4} + 2k® \; / k \in Z$$

$$\sin x - \cos x = -1 \Leftrightarrow x = 2k® \; / k \in Z \quad ou \quad x = \frac{-®}{2} + 2k® \; / k \in Z$$

بالتالي : $S = \left\{\frac{-®}{2} + 2k® \; / k \in Z\right\} \cup \{2k® \; / k \in Z\}$

لدينا :

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2 \Leftrightarrow \cos x - \sqrt{3}\sin x = 2\sqrt{3}$$

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2 \Leftrightarrow \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \sqrt{3}$$

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2 \Leftrightarrow \cos\left(\frac{®}{3}\right)\cos x - \sin\left(\frac{®}{3}\right)\sin x = \sqrt{3}$$

$$\frac{\cos x}{\sqrt{3}} - \sin x = 2 \Leftrightarrow \cos\left(x + \frac{®}{3}\right) = \sqrt{3}$$

بما أن $\sqrt{3} > 1$ فإن : $S = ¤$

تمرين 3 :

لدينا : $2\sin^2 x+3\cos x=3 \Leftrightarrow 2(1-\cos^2 x)+3\cos x=3 \Leftrightarrow 2\cos^2 x-3\cos x+1=0$

نضع : $t=\cos x$ منه : $2t^2-3t+1=0$ ، $\Delta=9-8=1$ منه : $t=\frac{3+1}{4}=1$ أو $t=\frac{3-1}{4}=\frac{1}{2}$

منه : $\cos x=1$ أو $\cos=\frac{1}{2}$ ، بالتالي : $S=\left\{\frac{\pi}{3}+2k\pi\ /\ k\in Z\right\}\cup\left\{-\frac{\pi}{3}+2k\pi\ /\ k\in Z\right\}\cup\left\{2k\pi\ /\ k\in Z\right\}$

بما أن $2+\sin x\geq 1>0$ و $2+\cos x\geq 1>0$ فمجموعة صلاحية المعادلة هي : IR

$$\frac{\sin x}{2+\cos x}=\frac{\cos x}{2+\sin x} \Leftrightarrow 2\sin x+\sin^2 x=2\cos x+\cos^2 x$$

$$\frac{\sin x}{2+\cos x}=\frac{\cos x}{2+\sin x} \Leftrightarrow 2(\sin x-\cos x)+\sin^2 x-\cos^2 x=0$$

$$\frac{\sin x}{2+\cos x}=\frac{\cos x}{2+\sin x} \Leftrightarrow (\sin x-\cos x)(2+\sin x+\cos x)=0$$

$$\frac{\sin x}{2+\cos x}=\frac{\cos x}{2+\sin x} \Leftrightarrow \cos x=\sin x \quad ou \quad \sin x+\cos x=-2$$

$$\frac{\sin x}{2+\cos x}=\frac{\cos x}{2+\sin x} \Leftrightarrow \cos x=\cos\left(\frac{\pi}{2}-x\right) \quad ou \quad \frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=\frac{-2}{\sqrt{2}}$$

$$\frac{\sin x}{2+\cos x}=\frac{\cos x}{2+\sin x} \Leftrightarrow \begin{array}{l} x=\frac{\pi}{2}-x+2k\pi\ /\ k\in Z \ ou \ \ x=\frac{-\pi}{2}+x+2k\pi\ /\ k\in Z \\ \quad ou \ \ \sin\left(\frac{\pi}{4}\right)\sin x+\cos\left(\frac{\pi}{4}\right)\cos x=-\sqrt{2}\end{array}$$

$$\frac{\sin x}{2+\cos x}=\frac{\cos x}{2+\sin x} \Leftrightarrow 2x=\frac{\pi}{2}+2k\pi\ /\ k\in Z \ ou \ \frac{\pi}{2}=2k\pi\ /\ k\in Z \ \ ou \ \ \underbrace{\cos\left(x-\frac{\pi}{4}\right)=-\sqrt{2}}_{impossible, car\ -\sqrt{2}<-1}$$

الآن لدينا :

$$\frac{\sin x}{2+\cos x}=\frac{\cos x}{2+\sin x} \Leftrightarrow x=\frac{\pi}{4}+k\pi\ /\ k\in Z \ ou \ \underbrace{k=\frac{1}{4}/\ k\in Z}_{impossible, car\ \frac{1}{4}\notin Z}$$

بالتالي : $S=\left\{\frac{\pi}{4}+k\pi\ /\ k\in Z\right\}$

مجموعة صلاحية المعادلة هي : $D=IR-\left\{\frac{\pi}{2}+k\pi\ /\ k\in Z\right\}$

لدينا في هذه المجموعة :

$$\tan x=\sin 2x \Leftrightarrow \frac{\sin x}{\cos x}=2\sin x\cos x \Leftrightarrow \sin x=2\sin x\cos^2 x \Leftrightarrow \sin x\left(1-2\cos^2 x\right)=0$$

$$\tan x=\sin 2x \Leftrightarrow \sin x=0 \quad ou \quad \cos x=\frac{\sqrt{2}}{2} \quad ou \quad \cos x=\frac{-\sqrt{2}}{2}$$

بالتالي:

$$S=\left\{k\pi\ /\ k\in Z\right\}\cup\left\{\frac{\pi}{4}+2k\pi\ /\ k\in Z\right\}\cup\left\{\frac{-\pi}{4}+2k\pi\ /\ k\in Z\right\}\cup\left\{\frac{3\pi}{4}+2k\pi\ /\ k\in Z\right\}\cup\left\{\frac{-3\pi}{4}+2k\pi\ /\ k\in Z\right\}$$

$$\tan x = \sin 2x \Leftrightarrow \frac{\sin x}{\cos x} = 2\sin x \cos x \Leftrightarrow \sin x = 2\sin x \cos^2 x \Leftrightarrow \sin x\left(1-2\cos^2 x\right)=0$$

أو أيضا :

$$\tan x = \sin 2x \Leftrightarrow -\sin x \cos(2x) = 0 \Leftrightarrow \sin x = 0 \quad ou \quad \cos 2x = 0$$

$$\tan x = \sin 2x \Leftrightarrow x = k\pi \,/\, k\in Z \quad ou \quad 2x = \frac{\pi}{2} + k\pi \,/\, k \in Z$$

$$\tan x = \sin 2x \Leftrightarrow x = k\pi \,/\, k\in Z \quad ou \quad x = \frac{\pi}{4} + \frac{k\pi}{2} \,/\, k \in Z$$

بالتالي : $S = \{k\pi \,/\, k\in Z\} \cup \left\{\frac{\pi}{4} + \frac{k\pi}{2} \,/\, k\in Z\right\}$

رغم أنه يبدو اختلاف حلي الطريقتين إلا أنهما في الحقيقة يمثلان نفس المجموعة

الطريقة الثانية أفضل لكنها تتطلب ملاحظة بعض الصيغ المثلثية الهامة : $\cos 2x = 2\cos^2 x - 1$

لدينا : $\cos 2x - 7\sin x = 4 \Leftrightarrow 1 - 2\sin^2 x - 7\sin x - 4 = 0 \Leftrightarrow 2\sin^2 x + 7\sin x + 3 = 0$

نضع : $t = \sin x$ فنجد: $2t^2 + 7t + 3 = 0$ ، $\Delta = 49 - 24 = 25$ منه : $t = \frac{-7+5}{4} = \frac{-1}{2}$ أو $t = \frac{-7-5}{4} = -3$

منه : $\cos x = -3$ (غير ممكن لأن $-3 < -1$) أو $\cos = \frac{-1}{2}$

بالتالي : $S = \left\{\frac{2\pi}{3} + 2k\pi \,/\, k\in Z\right\} \cup \left\{-\frac{2\pi}{3} + 2k\pi \,/\, k \in Z\right\}$

لدينا :

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow 2\cos(4x)\sin(x) = 2\cos(4x)\cos(2x)$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow \cos(4x)\left(\sin(x) - \cos(2x)\right) = 0$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow (\cos 4x = 0) \; ou \; (\cos 2x = \sin x)$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow (4x = 2k\pi \,/\, k\in Z) \; ou \; \left(\cos 2x = \cos\left(\frac{\pi}{2} - x\right)\right)$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow \left(x = \frac{k\pi}{2} \,/\, k\in Z\right) \; ou \; \left(2x = \frac{\pi}{2} - x + 2k\pi \,/\, k \in Z\right) \; ou \; \left(2x = \frac{-\pi}{2} + x + 2k\pi \,/\, k\in Z\right)$$

$$\sin 5x - \sin 3x = \cos 6x + \cos 2x \Leftrightarrow \left(x = \frac{k\pi}{2} \,/\, k\in Z\right) \; ou \; \left(x = \frac{\pi}{6} + \frac{2k\pi}{3} \,/\, k \in Z\right) \; ou \; \left(x = \frac{-\pi}{2} + 2k\pi \,/\, k\in Z\right)$$

بالتالي : $S = \left\{\frac{2\pi}{3} + 2k\pi \,/\, k\in Z\right\} \cup \left\{-\frac{2\pi}{3} + 2k\pi \,/\, k \in Z\right\}$

لدينا :

$$2\sin^2 x+\sqrt{3}\sin 2x=3 \Leftrightarrow 1-\cos 2x+\sqrt{3}\sin 2x=3$$
$$2\sin^2 x+\sqrt{3}\sin 2x=3 \Leftrightarrow \cos 2x-\sqrt{3}\sin 2x=-2$$
$$2\sin^2 x+\sqrt{3}\sin 2x=3 \Leftrightarrow \frac{1}{2}\cos 2x-\frac{\sqrt{3}}{2}\sin 2x=-1$$
$$2\sin^2 x+\sqrt{3}\sin 2x=3 \Leftrightarrow \cos\left(\frac{\pi}{3}\right)\cos 2x-\sin\left(\frac{\pi}{3}\right)\sin 2x=-1$$
$$2\sin^2 x+\sqrt{3}\sin 2x=3 \Leftrightarrow \cos\left(2x+\frac{\pi}{3}\right)=-1$$
$$2\sin^2 x+\sqrt{3}\sin 2x=3 \Leftrightarrow 2x+\frac{\pi}{3}=\pi+2k\pi \ / \ k\in Z$$
$$2\sin^2 x+\sqrt{3}\sin 2x=3 \Leftrightarrow x=\frac{\pi}{3}+k\pi \ / \ k\in Z$$

بالتالي : $S=\left\{\frac{\pi}{3}+k\pi \ / \ k\in Z\right\}$

لدينا :

$$\sqrt{2}\sin\left(x-\frac{\pi}{3}\right)-\sin x=\cos x \Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{3}\right)=\cos x+\sin x$$
$$\sqrt{2}\sin\left(x-\frac{\pi}{3}\right)-\sin x=\cos x \Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x$$
$$\sqrt{2}\sin\left(x-\frac{\pi}{3}\right)-\sin x=\cos x \Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{4}\right)\cos x+\cos\left(\frac{\pi}{4}\right)\sin x$$
$$\sqrt{2}\sin\left(x-\frac{\pi}{3}\right)-\sin x=\cos x \Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\sin\left(x+\frac{\pi}{4}\right)$$
$$\sqrt{2}\sin\left(x-\frac{\pi}{3}\right)-\sin x=\cos x \Leftrightarrow x-\frac{\pi}{3}=x+\frac{\pi}{4}+2k\pi \ / \ k\in Z \ \ ou \ \ x-\frac{\pi}{3}=\pi-\left(x+\frac{\pi}{4}\right)+2k\pi \ / \ k\in Z$$
$$\sqrt{2}\sin\left(x-\frac{\pi}{3}\right)-\sin x=\cos x \Leftrightarrow -\frac{7\pi}{12}=2k\pi \ / \ k\in Z \ \ ou \ \ 2x=\frac{13\pi}{12}+2k\pi \ / \ k\in Z$$
$$\sqrt{2}\sin\left(x-\frac{\pi}{3}\right)-\sin x=\cos x \Leftrightarrow k=\frac{7}{24} \ / \ k\in Z \ \ ou \ \ x=\frac{13\pi}{24}+k\pi \ / \ k\in Z$$

بالتالي : $S=\left\{\frac{13\pi}{24}+k\pi \ / \ k\in Z\right\}$

تمرين 4 :

1

لدينا لكل $(a,b)\in IR^2$ ، نضع : $A=\cos(a+b)\sin(a-b)$

$$A=(\cos a\cos b-\sin a\sin b)(\sin a\cos b-\cos a\sin b)$$
$$=\cos a\cos b\sin a\cos b-\cos a\cos b\cos a\sin b-\sin a\sin b\sin a\cos b+\sin a\sin b\cos a\sin b$$
$$=\sin a\cos a\cos^2 b-\sin b\cos b\cos^2 a-\sin b\cos b\sin^2 a+\sin a\cos a\sin^2 b$$
$$=\sin a\cos a(\cos^2 b+\sin^2 b)-\sin b\cos b(\cos^2 a+\sin^2 a)$$
$$A=\sin a\cos a-\sin b\cos b$$

2

لدينا حسب السؤال السابق :

$$\cos\left(x+\frac{\pi}{4}\right)\times\sin\left(x-\frac{\pi}{4}\right)=-\frac{1}{4}\iff \sin x\cos x-\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)=-\frac{1}{4}$$

$$\cos\left(x+\frac{\pi}{4}\right)\times\sin\left(x-\frac{\pi}{4}\right)=-\frac{1}{4}\iff \frac{1}{2}\sin 2x-\frac{1}{2}=-\frac{1}{4}$$

$$\cos\left(x+\frac{\pi}{4}\right)\times\sin\left(x-\frac{\pi}{4}\right)=-\frac{1}{4}\iff \frac{1}{2}\sin 2x=\frac{1}{4}$$

$$\cos\left(x+\frac{\pi}{4}\right)\times\sin\left(x-\frac{\pi}{4}\right)=-\frac{1}{4}\iff \sin 2x=\frac{1}{2}$$

$$\cos\left(x+\frac{\pi}{4}\right)\times\sin\left(x-\frac{\pi}{4}\right)=-\frac{1}{4}\iff 2x=\frac{\pi}{6}+2k\pi\ /\ k\in Z\quad ou\quad 2x=\frac{5\pi}{6}+2k\pi\ /\ k\in Z$$

$$\cos\left(x+\frac{\pi}{4}\right)\times\sin\left(x-\frac{\pi}{4}\right)=-\frac{1}{4}\iff x=\frac{\pi}{12}+k\pi\ /\ k\in Z\quad ou\quad x=\frac{5\pi}{12}+k\pi\ /\ k\in Z$$

بالتالي: $S=\left\{\frac{\pi}{12}+k\pi\ /\ k\in Z\right\}\cup\left\{\frac{5\pi}{12}+k\pi\ /\ k\in Z\right\}$

تمرين 5 : نعتبر المعادلة : $(E):\ \sqrt{3}\sin(x)+\cos x=1$

1

لدينا :

$$(E)\iff \frac{\sqrt{3}}{2}\sin(x)+\frac{1}{2}\cos x=\frac{1}{2}$$

$$(E)\iff \sin\left(\frac{\pi}{3}\right)\sin(x)+\cos\left(\frac{\pi}{3}\right)\cos x=\cos\left(\frac{\pi}{3}\right)$$

$$(E)\iff \cos\left(x-\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)$$

$$(E)\iff x-\frac{\pi}{3}=\frac{\pi}{3}+2k\pi\ /\ k\in Z\quad ou\quad x-\frac{\pi}{3}=\frac{-\pi}{3}+2k\pi\ /\ k\in Z$$

$$(E)\iff x=\frac{2\pi}{3}+2k\pi\ /\ k\in Z\quad ou\quad x=2k\pi\ /\ k\in Z$$

بالتالي: $S=\left\{\frac{2\pi}{3}+2k\pi\ /\ k\in Z\right\}\cup\left\{2k\pi\ /\ k\in Z\right\}$

2

لدينا :

$$\frac{\sqrt{3}}{2}\sin(x_k)+\frac{1}{2}\cos(x_k)=\frac{\sqrt{3}}{2}\sin(\pi+2k\pi)+\frac{1}{2}\cos(\pi+2k\pi)=\frac{\sqrt{3}}{2}\sin(\pi)+\frac{1}{2}\cos(\pi)=\frac{-1}{2}$$

إذن $x_k=\pi+2k\pi$ ليس حلا للمعادلة (E) .

3

بما أن أي حل لمعادلة يحقق : $x\neq\pi+2k\pi$ فإن : $\frac{x}{2}\neq\frac{\pi}{2}+k\pi$ لكل $k\in Z$

إذن يمكننا أن نضع : $t=\tan\frac{x}{2}$ ، لدينا الآن : $\sin x=\frac{2t}{1+t^2}$ و $\cos x=\frac{1-t^2}{1+t^2}$

منه :

$$(E)\iff \frac{\sqrt{3}}{2}\frac{2t}{1+t^2}+\frac{1}{2}\frac{1-t^2}{1+t^2}=\frac{1}{2}$$

$$(E)\iff 2\sqrt{3}t+1-t^2=1+t^2$$

$$(E)\iff 2t^2-2\sqrt{3}t=0$$

$$(E)\iff t^2-\sqrt{3}t=0$$

	الآن لدينا : $t=0$ أو $t=\sqrt{3}$ منه : $\tan\left(\frac{x}{2}\right)=0$ أو $\tan\left(\frac{x}{2}\right)=\sqrt{3}$ منه : $\frac{x}{2}=k\pi \,/\, k\in Z$ أو $\frac{x}{2}=\frac{\pi}{3}+k\pi \,/\, k\in Z$ منه : $x=2k\pi \,/\, k\in Z$ أو $x=\frac{2\pi}{3}+2k\pi \,/\, k\in Z$ بالتالي : $S=\left\{\frac{2\pi}{3}+2k\pi \,/\, k\in Z\right\}\cup\left\{2k\pi \,/\, k\in Z\right\}$	4