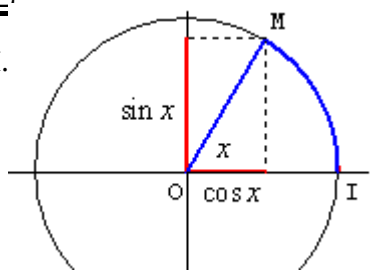
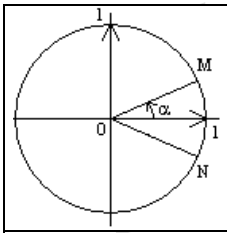
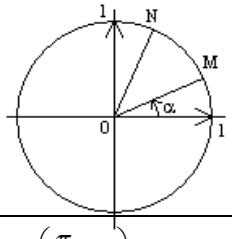
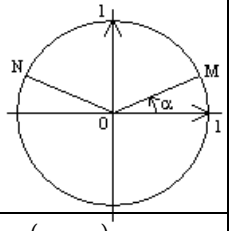
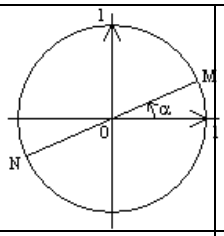
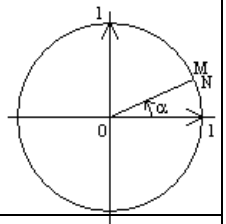
FORMULAIRE DE TRIGONOMETRIE



x étant une mesure de l'angle $(\overrightarrow{OI};\overrightarrow{OM})$, cos x est l'abscisse de M, sin x est l'ordonnée de M.

$\begin{cases}\cos 0=1\\ \sin 0=0\end{cases}$ $\quad\begin{cases}\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}\\ \sin\frac{\pi}{6}=\frac{1}{2}\end{cases}$ $\quad\begin{cases}\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}\\ \sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}\end{cases}$ $\quad\begin{cases}\cos\frac{\pi}{3}=\frac{1}{2}\\ \sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}\end{cases}$ $\quad\begin{cases}\cos\frac{\pi}{2}=0\\ \sin\frac{\pi}{2}=1\end{cases}$

Pour tout réel α, $\boxed{\cos^2\alpha+\sin^2\alpha=1}$

				
$\begin{cases}\cos(-\alpha)=\cos\alpha\\ \sin(-\alpha)=-\sin\alpha\end{cases}$	$\begin{cases}\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha\\ \sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha\end{cases}$ α et $\frac{\pi}{2}-\alpha$ sont <u>complémentaires.</u> leur somme vaut $\frac{\pi}{2}$	$\begin{cases}\cos(\pi-\alpha)=-\cos\alpha\\ \sin(\pi-\alpha)=\sin\alpha\end{cases}$ α et $\pi-\alpha$ sont <u>supplémentaires</u> leur somme vaut π	$\begin{cases}\cos(\alpha+\pi)=-\cos\alpha\\ \sin(\alpha+\pi)=-\sin\alpha\end{cases}$	$\begin{cases}\cos(\alpha+2\pi)=\cos\alpha\\ \sin(\alpha+2\pi)=\sin\alpha\end{cases}$

<u>Formules de transformation et de dupplication :</u>

$\cos(a+b)=\cos a\cdot\cos b-\sin a\cdot\sin b$ $\qquad$ $\sin(a+b)=\sin a\cdot\cos b+\sin b\cdot\cos a$

$\cos(a-b)=\cos a\cdot\cos b+\sin a\cdot\sin b$ $\qquad$ $\sin(a-b)=\sin a\cdot\cos b-\sin b\cdot\cos a$

$\cos(2a)=\begin{cases}\cos^2 a-\sin^2 a\\ 2\cos^2 a-1\\ 1-2\sin^2 a\end{cases}$ $\qquad$ $\sin(2a)=2\cdot\sin a\cdot\cos a$ $\qquad$ $\tan(2a)=\frac{2\cdot\tan a}{1-\tan^2 a}$

$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\cdot\tan b}$ et $\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\cdot\tan b}$

<u>Transformation de produit en somme</u>

$\cos a\cdot\cos b=\frac{1}{2}\left(\cos(a+b)+\cos(a-b)\right)$ $\qquad$ $\sin a\cdot\sin b=\frac{1}{2}\left(\cos(a-b)-\cos(a+b)\right)$ $\qquad$ $\sin a\cdot\cos b=\frac{1}{2}\left(\sin(a+b)+\sin(a-b)\right)$

<u>Transformation de somme en produit</u>

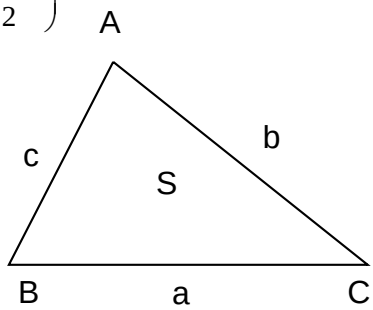
$\cos p+\cos q=2\cdot\cos\left(\frac{p+q}{2}\right)\cdot\cos\left(\frac{p-q}{2}\right)$ $\qquad$ $\cos p-\cos q=-2\cdot\sin\left(\frac{p+q}{2}\right)\cdot\sin\left(\frac{p-q}{2}\right)$

$\sin p+\sin q=2\cdot\sin\left(\frac{p+q}{2}\right)\cdot\cos\left(\frac{p-q}{2}\right)$ $\qquad$ $\sin p-\sin q=2\cdot\sin\left(\frac{p-q}{2}\right)\cdot\cos\left(\frac{p+q}{2}\right)$

<u>Formules utilisant des tangentes :</u>

$\cos^2 x=\frac{1}{1+\tan^2 x}=\frac{1}{2}\left(1+\cos(2x)\right)$ $\qquad$ $\sin^2 x=\frac{\tan^2 x}{1+\tan^2 x}=\frac{1}{2}\left(1-\cos(2x)\right)$

Posons $t=\tan\frac{x}{2}$, alors $\cos x=\frac{1-t^2}{1+t^2}$, $\sin x=\frac{2t}{1+t^2}$ et $\tan x=\frac{2t}{1-t^2}$



<u>Relations métriques dans le triangle.</u>

Formule d'Al-Kashi	$a^2=b^2+c^2-2bc\cos\left(\widehat{A}\right)$, $b^2=a^2+c^2-2ac\cos\left(\widehat{B}\right)$ et $c^2=a^2+b^2-2ab\cos\left(\widehat{C}\right)$		
Aire du triangle	$S=\frac{1}{2}ab\sin\widehat{C}=\frac{1}{2}bc\sin\hat{a}=\frac{1}{2}ac\sin\widehat{B}$. Enfin	$\boxed{\frac{a}{\sin\widehat{A}}=\frac{b}{\sin\widehat{B}}=\frac{c}{\sin\widehat{C}}}$	(formule des 3 sinus)