جيا 6 بنشواف .

التمرين الأول :

-1)

$$\lim_{x\to 3}\frac{2x^2-3x-9}{x^2+x-12}=\lim_{x\to 3}\frac{(x-3)(2x+3)}{(x-3)(x+4)}$$

$$=\lim_{x\to 3}\frac{2x+3}{x+4}$$

$$=\boxed{\frac{9}{7}}$$

-2)

$$\lim_{x\to -3}\frac{\sqrt{3-2x}-3}{\sqrt{-3x}-3}=\lim_{x\to -3}\frac{(\sqrt{3-2x}-3)(\sqrt{3-2x}+3)(\sqrt{-3x}+3)}{(\sqrt{-3x}-3)(\sqrt{-3x}+3)(\sqrt{3-2x}+3)}$$

$$=\lim_{x\to -3}\frac{(3-2x-9)(\sqrt{-3x}+3)}{(-3x-9)(\sqrt{3-2x}+3)}$$

$$=\lim_{x\to -3}\frac{(-2x-6)(\sqrt{-3x}+3)}{(-3x-9)(\sqrt{3-2x}+3)}$$

$$=\lim_{x\to -3}\frac{-2(x+3)(\sqrt{-3x}+3)}{-3(x+3)(\sqrt{3-2x}+3)}$$

$$=\lim_{x\to -3}\frac{2}{3}\times\frac{\sqrt{-3x}+3}{\sqrt{3-2x}+3}$$

$$=\frac{2}{3}\times\frac{6}{6}=\boxed{\frac{2}{3}}$$

-3)

$$\lim_{\substack{x\to -5\\ x>-5}}\frac{x^2-5}{x^2+5x}.$$

لدينا $\lim_{x\to -5} x^2-5=20$

و $\lim_{x\to -5} x^2+5x=0$

جدول إشارة المقام :

x	$-\infty$	-5		0	$+\infty$
x^2+5x	$+$	0	$-$	0	$+$

ومما سبق فإن : $\lim_{\substack{x\to -5\\ x>-5}}\frac{x^2-5}{x^2+5x}=-\infty$

(4 -

$$\lim_{x\to+\infty} \sqrt{4x^2+2x}-2x = \lim_{x\to+\infty} \frac{4x^2+2x-4x^2}{\sqrt{4x^2+2x}+2x}$$

$$= \lim_{x\to+\infty} \frac{2x}{x\sqrt{4+\frac{2}{x}}+2x}$$

$$= \lim_{x\to+\infty} \frac{2x}{x(\sqrt{4+\frac{2}{x}}+2)}$$

$$= \lim_{x\to+\infty} \frac{2}{\sqrt{4+\frac{2}{x}}+2}$$

لدينا $\lim_{x\to+\infty} \frac{2}{x} = 0$ يعني أن $\lim_{x\to+\infty} \sqrt{4+\frac{2}{x}} = 2$ إذن $\lim_{+\infty} \sqrt{4+\frac{2}{x}}+2 = 4$

ومنه فإن $\lim_{x\to+\infty} \frac{2}{\sqrt{4+\frac{2}{x}}+2} = \frac{2}{4} = \boxed{\frac{1}{2}}$

التمرين الثاني:

(1 - $f(x) = x^3 - x^2$ و $a = 2$

$$f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2} \frac{x^3-x^2-4}{x-2}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+x+2)}{\cancel{(x-2)}}$$

$$f'(2) = \lim_{x\to 2} x^2+x+2 = \boxed{8}$$

إذن f تقبل الاشتقاق في $a=2$ و $f'(2)=8$.

(2 - $f(x) = \sqrt{3x+4}-3$ و $a=-1$

$$f'(-1) = \lim_{x\to -1} \frac{f(x)-f(-1)}{x+1}$$

$$= \lim_{x\to -1} \frac{\sqrt{3x+4}-3+2}{x+1}$$

$$= \lim_{x\to -1} \frac{\sqrt{3x+4}-1}{x+1}$$

$$= \lim_{x\to -1} \frac{(\sqrt{3x+4}-1)(\sqrt{3x+4}+1)}{(x+1)(\sqrt{3x+4}+1)}$$

$$f'(-1) = \lim_{x \to -1} \frac{3x+3}{(x+1)(\sqrt{3x+4}+1)}$$

$$= \lim_{x \to -1} \frac{3\cancel{(x+1)}}{\cancel{(x+1)}(\sqrt{3x+4}+1)}$$

$$f'(-1) = \boxed{\frac{3}{2}}$$

إذن f تقبل الاشتقاق في $a=-1$ و $f'(-1)=\frac{3}{2}$

3- $f(x) = \frac{x^2}{3x-2}$ و $a = 1$

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{\frac{x^2}{3x-2} - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{\frac{x^2-3x+2}{3x-2}}{x-1}$$

$$= \lim_{x \to 1} \frac{\frac{(x-1)(x-2)}{3x-2}}{(x-1)}$$

$$f'(1) = \lim_{x \to 1} \frac{x-2}{3x-2} = \boxed{-1}$$

إذن f تقبل الاشتقاق في $a=1$ و $f'(1) = -1$

التمرين الثالث:

1- $f(x) = x^2 - \frac{4}{3}x\sqrt{x} + 2$

$$f'(x) = (x^2)' - \frac{4}{3}(x\sqrt{x})'$$

$$= 2x - \frac{4}{3}\left[(x)'\sqrt{x} + x\sqrt{x}'\right]$$

$$= 2x - \frac{4}{3}\left[\sqrt{x} + \frac{x}{2\sqrt{x}}\right]$$

$$= 2x - \frac{4}{3} \cdot \frac{3}{2}\sqrt{x}$$

$$= 2x - 2\sqrt{x}$$

$$\boxed{f'(x) = 2\sqrt{x}(\sqrt{x}-1)}$$

(2 -

$$f(x) = (x+2)\sqrt{x-3} + \sqrt{3}$$

$$f'(x) = (x+2)'\left(\sqrt{x-3}\right) + \left(\sqrt{x-3}\right)'(x+2)$$

$$= \sqrt{x-3} + \frac{x+2}{2\sqrt{x-3}}$$

$$= \frac{2(x-3)+(x+2)}{2\sqrt{x-3}}$$

$$f'(x) = \frac{2x-6+x+2}{2\sqrt{x-3}}$$

$$\boxed{f'(x) = \frac{3x-4}{2\sqrt{x-3}}}$$

(4 -

$$f(x) = \frac{x+2}{\sqrt{x-1}+1}$$

$$f'(x) = \frac{(x+2)'\left(\sqrt{x-1}+1\right) - (x+2)\left(\sqrt{x-1}+1\right)'}{\left(\sqrt{x-1}+1\right)^2}$$

$$f'(x) = \frac{\left(\sqrt{x-1}+1\right) - (x+2)\left(\frac{1}{2\sqrt{x-1}}\right)}{\left(\sqrt{x-1}+1\right)^2}$$

$$= \frac{\frac{2x-2+2\sqrt{x-1}-(x+2)}{2\sqrt{x-1}}}{\left(\sqrt{x-1}+1\right)^2}$$

$$f'(x) = \frac{x+2\sqrt{x-1}-4}{2\left(\sqrt{x-1}+1\right)^2\sqrt{x-1}}$$

لدينا

$$\left(\sqrt{x-1}-1\right)\left(\sqrt{x-1}+3\right) = x-1+3\sqrt{x-1}-\sqrt{x-1}-3$$

$$\left(\sqrt{x-1}-1\right)\left(\sqrt{x-1}+3\right) = x+2\sqrt{x-1}-4.$$

ومنه فإن

$$f'(x) = \frac{\left(\sqrt{x-1}-1\right)\left(\sqrt{x-1}+3\right)}{2\left(\sqrt{x-1}+1\right)^2\sqrt{x-1}}.$$

(3 -

$$f(x) = \frac{2x^3-3x^2}{(x+1)^2}$$

$$f'(x) = \frac{(2x^3-3x^2)'(x+1)^2 - \left((x+1)^2\right)'(2x^3-3x^2)}{(x+1)^4}$$

$$= \frac{(6x^2-6x)(x+1)^2 - 2(x+1)(2x^3-3x^2)}{(x+1)^4}$$

$$f'(x)=\frac{(x+1)\left[6x(x-1)(x+1)-4x^3+6x^2\right]}{(x+1)^4}$$

$$f'(x)=\frac{6x(x^2-1)-4x^3+6x^2}{(x+1)^3}$$

$$=\frac{6x^3-6x-4x^3+6x^2}{(x+1)^3}$$

$$=\frac{2x^3+6x^2-6x}{(x+1)^3}$$

$$\boxed{f'(x)=\frac{2x(x^2+3x-3)}{(x+1)^3}}$$

التمرين الرابع:

1- أ- لنبين أن $U_n>0$:

لدينا من أجل $n=0$ ، $U_0=2>0$

- نفترض أن $U_n>0$ ونبين أن $U_{n+1}>0$.

لدينا $U_{n+1}=\frac{U_n}{3+2U_n}$ ⟸ لدينا $U_n>0$ ومنه فإن $3+2U_n>3>0$

إذن $U_{n+1}=\frac{U_n}{3+2U_n}>0$ وبالتالي فإن $U_n>0$.

ب- لنبين أن $(U_n)_n$ تناقصية:

لدينا $U_{n+1}-U_n=\frac{U_n}{3+2U_n}-U_n$

$$=\frac{U_n-3U_n-2U_n^2}{3+2U_n}$$

$$=\frac{-2U_n^2-4U_n}{3+2U_n}$$

$$U_{n+1}-U_n=\frac{-2U_n(U_n+2)}{3+2U_n}$$

لدينا $3+2U_n>0$ و $-2U_n<0$ و $U_n+2>2>0$ أي أن $-2U_n(U_n+2)<0$

ومنه: $U_{n+1}-U_n=\frac{-2U_n(U_n+2)}{3+2U_n}<0$

إذن $(U_n)_n$ متتالية تناقصية.

2 -

أ - لنبين أن v_n متتالية هندسية أساسها $q = \frac{1}{3}$ وحدها الأول $v_0 = \frac{2}{3}$

لدينا $v_n = \frac{u_n}{u_n + 1} \Leftarrow v_{n+1} = \frac{u_{n+1}}{u_{n+1} + 1}$

يعني $v_n = \dfrac{\frac{u_n}{3 + 2u_n}}{\frac{u_n}{3 + 2u_n} + 1}$

يعني $v_n = \dfrac{\frac{u_n}{3 + 2u_n}}{\frac{u_n + 3 + 2u_n}{3 + 2u_n}}$

يعني $v_n = \frac{u_n}{3(u_n + 1)} \Leftarrow v_n = \frac{1}{3} \times v_{n+1}$

إذن v_n متتالية هندسية أساسها $\frac{1}{3}$

ولدينا $v_0 = \frac{u_0}{u_0 + 1} \Leftarrow v_0 = \frac{2}{2+1} \Leftarrow \boxed{v_0 = \frac{2}{3}}$

ب - لنحدد v_n بدلالة n ونبين أن $u_n = \frac{2}{3^{n+1} - 2}$

لدينا $v_n = v_0 \cdot \left(\frac{1}{3}\right)^n$

يعني $v_n = \frac{2}{3} \cdot \left(\frac{1}{3}\right)^n$ إذن $\boxed{v_n = \frac{2}{3^{n+1}}}$

ولدينا $v_n = \frac{u_n}{u_n + 1}$ إذن $v_n u_n + v_n - u_n = 0$

يعني $u_n (v_n - 1) + v_n = 0$

يعني $u_n = \frac{v_n}{1 - v_n}$

ومنه فإن $u_n = \dfrac{\frac{2}{3^{n+1}}}{1 - \frac{2}{3^{n+1}}}$

إذن $\boxed{u_n = \frac{2}{3^{n+1} - 2}}$

3 - لنبين أن $S = n + \frac{1}{3^{n+1}}$

لدينا $S_n = \frac{1}{u_0 + 1} + \frac{1}{u_1 + 1} + \cdots + \frac{1}{u_n + 1}$

ولدينا $U_n = \dfrac{V_n}{1-V_n}$

يعني $U_n + 1 = \dfrac{V_n + 1 - V_n}{1 - V_n}$

يعني أن $\dfrac{1}{U_n+1} = \dfrac{1-V_n}{1} \Leftrightarrow \boxed{\dfrac{1}{U_n+1} = 1 - V_n}$

نعوض:

$$S_n = 1-V_0 + 1 - V_1 + \dots + 1 - V_n$$

$$S_n = (1+1+\dots+1) - (V_0 + V_1 + \dots + V_n)$$

$$S_n = (n+1) - V_0 \times \frac{1-q^{n+1}}{1-q}$$

$$S_n = n-1 - \frac{2}{3} \times \frac{1 - \frac{1}{3^{n+1}}}{\frac{2}{3}}$$

$$S_n = n+1 - 1 + \frac{1}{3^{n+1}}$$

$$\boxed{S_n = n + \frac{1}{3^{n+1}}}$$