

سلسلة 3

الحساب المثلثي
حلول مقترحة

السنة 1 بكالوريا علوم تجريبية

تمرين 1 : المعادلات الموجودة بالتمرين كلها على شكل: $a \cos x + b \sin x = c$ ، حيث نعمل ونختزل بـ $r = \sqrt{a^2 + b^2}$

لنحل المعادلة: $\cos x - \sin x = \sqrt{2}$

$$\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = 1$$

$$\cos\left(\frac{\pi}{4}\right) \cos x - \sin\left(\frac{\pi}{4}\right) \sin x = 1$$

$$S = \left\{ \frac{-\pi}{4} + 2k\pi / k \in \mathbb{Z} \right\}$$

بالتالي:

$$\cos\left(\frac{\pi}{4} + x\right) = 1$$

لدينا:

$$x + \frac{\pi}{4} = 2k\pi / k \in \mathbb{Z}$$

$$x = \frac{-\pi}{4} + 2k\pi / k \in \mathbb{Z}$$

لنحل المعادلة: $\cos x - \sqrt{3} \sin x = 1$

$$\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3}\right) \cos x - \sin\left(\frac{\pi}{3}\right) \sin x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3} + x\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$x + \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z} \text{ ou } x + \frac{\pi}{3} = \frac{-\pi}{3} + 2k\pi / k \in \mathbb{Z}$$

$$x = 2k\pi / k \in \mathbb{Z} \text{ ou } x = \frac{-2\pi}{3} + 2k\pi / k \in \mathbb{Z}$$

$$S = \{2k\pi / k \in \mathbb{Z}\} \cup \left\{ \frac{-2\pi}{3} + 2k\pi / k \in \mathbb{Z} \right\}$$

لنحل المعادلة: $\sqrt{3} \cos x - \sin x = \sqrt{2}$

$$\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right) \cos x - \sin\left(\frac{\pi}{4}\right) \sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{4}\right)$$

لدينا:

$$x + \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi / k \in \mathbb{Z} \text{ ou } x + \frac{\pi}{4} = \frac{-\pi}{4} + 2k\pi / k \in \mathbb{Z}$$

$$x = \frac{\pi}{3} - \frac{\pi}{4} + 2k\pi / k \in \mathbb{Z} \text{ ou } x = \frac{-\pi}{3} - \frac{\pi}{4} + 2k\pi / k \in \mathbb{Z}$$

$$x = \frac{\pi}{12} + 2k\pi / k \in \mathbb{Z} \text{ ou } x = \frac{-7\pi}{12} + 2k\pi / k \in \mathbb{Z}$$

$$S = \left\{ \frac{\pi}{12} + 2k\pi / k \in \mathbb{Z} \right\} \cup \left\{ \frac{-7\pi}{12} + 2k\pi / k \in \mathbb{Z} \right\}$$

بالتالي:

لنحل المعادلة: $\cos \frac{x}{2} - \sin \frac{x}{2} = -1$

$$\frac{1}{\sqrt{2}} \cos \frac{x}{2} - \frac{1}{\sqrt{2}} \sin \frac{x}{2} = -1$$

$$\cos\left(\frac{\pi}{4}\right) \cos \frac{x}{2} - \sin\left(\frac{\pi}{4}\right) \sin \frac{x}{2} = -1$$

$$\cos\left(\frac{\pi}{4} + \frac{x}{2}\right) = -1$$

$$S = \left\{ \frac{3\pi}{2} + 2k\pi / k \in Z \right\} \text{ بالتالي}$$

لدينا:

$$\frac{x}{2} + \frac{\pi}{4} = \pi + k\pi / k \in Z$$

$$\frac{x}{2} = \frac{3\pi}{4} + k\pi / k \in Z$$

$$x = \frac{3\pi}{2} + 2k\pi / k \in Z$$

لنحل المعادلة: $\sin x + \cos x = \sqrt{2}$

$$\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = 1$$

$$\cos\left(\frac{\pi}{4}\right) \cos x + \sin\left(\frac{\pi}{4}\right) \sin x = 1$$

$$S = \left\{ \frac{\pi}{4} + 2k\pi / k \in Z \right\} \text{ بالتالي}$$

$$\cos\left(\frac{\pi}{4} - x\right) = 1$$

لدينا:

$$x - \frac{\pi}{4} = 2k\pi / k \in Z$$

$$x = \frac{\pi}{4} + 2k\pi / k \in Z$$

لنحل المعادلة: $\sin x + \cos x = 1$

$$\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi}{4}\right) \cos x + \sin\left(\frac{\pi}{4}\right) \sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4} - x\right) = \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right)$$

لدينا:

$$\frac{\pi}{4} - x = \frac{\pi}{4} + 2k\pi / k \in Z \text{ ou } \frac{\pi}{4} - x = \frac{-\pi}{4} + 2k\pi / k \in Z$$

$$x = -2k\pi / k \in Z \text{ ou } x = \frac{\pi}{2} + 2k\pi / k \in Z$$

$$S = \{-2k\pi / k \in Z\} \cup \left\{ \frac{\pi}{2} + 2k\pi / k \in Z \right\} \text{ بالتالي}$$

$$\cos a = \cos b \Leftrightarrow a = b + 2k\pi \text{ ou } a = -b + 2k\pi / k \in Z$$

$$\cos a = 1 \Leftrightarrow a = 2k\pi / k \in Z ; \cos a = -1 \Leftrightarrow a = \pi + 2k\pi / k \in Z ; \cos a = 0 \Leftrightarrow a = \frac{\pi}{2} + k\pi / k \in Z$$

$$\sin a = \sin b \Leftrightarrow a = b + 2k\pi / k \in Z \text{ ou } a = \pi - b + 2k\pi / k \in Z$$

$$\sin a = 1 \Leftrightarrow a = \frac{\pi}{2} + 2k\pi / k \in Z ; \sin a = -1 \Leftrightarrow a = \frac{-\pi}{2} + 2k\pi / k \in Z ; \sin a = 0 \Leftrightarrow a = k\pi / k \in Z$$

$$f(x) = \frac{\cos x + \sin x}{\cos x - \sin x} : \text{تمرين 2}$$

لنحل المعادلة: $\sin x - \cos x = 0$

$$\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = 0$$

$$\cos\left(\frac{\pi}{4}\right) \cos x - \sin\left(\frac{\pi}{4}\right) \sin x = 0$$

$$S = \left\{ \frac{\pi}{4} + k\pi / k \in Z \right\} : \text{بالتالي}$$

$$\cos\left(\frac{\pi}{4} + x\right) = 0 : \text{لدينا}$$

$$\frac{\pi}{4} + x = \frac{\pi}{2} + k\pi / k \in Z$$

$$x = \frac{\pi}{4} + k\pi / k \in Z$$

1

$$Df = \{x \in IR / \cos x - \sin x \neq 0\} = IR_{-\left\{\frac{\pi}{4} + k\pi / k \in Z\right\}} : \text{لدينا} \quad f(x) = \frac{\cos x + \sin x}{\cos x - \sin x} \quad \text{إذن حسب السؤال السابق}$$

2

$$f(x) = \frac{(\cos x + \sin x)}{(\cos x - \sin x)} = \frac{(\cos x + \sin x)(\cos x + \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$f(x) = \frac{(\cos x + \sin x)^2}{\cos^2 x - \sin^2 x} = \frac{\cos^2 x + \sin^2 x + 2 \sin x \cos x}{\cos x \cos x - \sin x \sin x} = \frac{1 + \sin 2x}{\cos(x+x)} = \frac{1 + \sin 2x}{\cos 2x} : \text{لدينا}$$

3

لنحل المعادلة: $f(x) - \sqrt{3} = 0$ ، لدينا :

$$f(x) = \sqrt{3} \Leftrightarrow \frac{1 + \sin 2x}{\cos 2x} = \sqrt{3} \Leftrightarrow 1 + \sin 2x = \sqrt{3} \cos 2x \Leftrightarrow \sqrt{3} \cos 2x - \sin 2x = 1$$

$$\Leftrightarrow \frac{\sqrt{3}}{2} \cos 2x - \frac{1}{2} \sin 2x = \frac{1}{2} \Leftrightarrow \cos\left(\frac{\pi}{6}\right) \cos 2x - \sin\left(\frac{\pi}{6}\right) \sin 2x = \frac{1}{2}$$

$$\Leftrightarrow \cos\left(\frac{\pi}{6} + 2x\right) = \cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow \frac{\pi}{6} + 2x = \frac{\pi}{3} + 2k\pi / k \in Z \quad \text{ou} \quad \frac{\pi}{6} + 2x = \frac{-\pi}{3} + 2k\pi / k \in Z$$

$$\Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi / k \in Z \quad \text{ou} \quad 2x = \frac{-\pi}{2} + 2k\pi / k \in Z$$

$$f(x) = \sqrt{3} \Leftrightarrow x = \frac{\pi}{12} + k\pi / k \in Z \quad \text{ou} \quad x = \frac{-\pi}{4} + k\pi / k \in Z$$

$$S = \left\{ \frac{\pi}{12} + k\pi / k \in Z \right\} \cup \left\{ \frac{-\pi}{4} + k\pi / k \in Z \right\} : \text{بالتالي}$$

4

$$A(x) = \sqrt{3} \cos\left(2x - \frac{\pi}{3}\right) - \sin\left(2x - \frac{\pi}{3}\right) : \text{تمرين 3}$$

$$A(x) = \sqrt{3} \cos\left(2x - \frac{\pi}{3}\right) - \sin\left(2x - \frac{\pi}{3}\right) = 2 \left(\frac{\sqrt{3}}{2} \cos\left(2x - \frac{\pi}{3}\right) - \frac{1}{2} \sin\left(2x - \frac{\pi}{3}\right) \right)$$

$$= 2 \left(\cos\left(\frac{\pi}{6}\right) \cos\left(2x - \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right) \sin\left(2x - \frac{\pi}{3}\right) \right)$$

$$A(x) = 2 \cos\left(\frac{\pi}{6} + 2x - \frac{\pi}{3}\right) = 2 \cos\left(2x - \frac{\pi}{6}\right)$$

1

$$A(x) = 1 \Leftrightarrow 2 \cos\left(2x - \frac{\pi}{6}\right) = 1 \Leftrightarrow \cos\left(2x - \frac{\pi}{6}\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow 2x - \frac{\pi}{6} = \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z} \text{ ou } 2x - \frac{\pi}{6} = \frac{-\pi}{3} + 2k\pi / k \in \mathbb{Z}$$

$$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi / k \in \mathbb{Z} \text{ ou } 2x = \frac{-\pi}{6} + 2k\pi / k \in \mathbb{Z}$$

$$A(x) = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi / k \in \mathbb{Z} \text{ ou } x = \frac{-\pi}{12} + k\pi / k \in \mathbb{Z}$$

لتحديد الحلول الموجودة في المجال $]-\pi; \pi]$ نؤطر العدد النسبي k ونحدد قيمته:

$$\frac{\pi}{4} + k\pi \in]-\pi; \pi] \Leftrightarrow -\pi < \frac{\pi}{4} + k\pi \leq \pi \Leftrightarrow -4\pi < \pi + 4k\pi \leq 4\pi \Leftrightarrow -5\pi < 4k\pi \leq 3\pi$$

$$\Leftrightarrow -5 < 4k \leq 3 \Leftrightarrow \frac{-5}{4} < k \leq \frac{3}{4} \Leftrightarrow k = -1 \text{ ou } k = 0$$

2

$$\frac{\pi}{4} + k\pi \in]-\pi; \pi] \Leftrightarrow x = \frac{\pi}{4} - \pi = \frac{-3\pi}{4} \text{ ou } x = \frac{\pi}{4}$$

$$\frac{-\pi}{12} + k\pi \in]-\pi; \pi] \Leftrightarrow -\pi < \frac{-\pi}{12} + k\pi \leq \pi \Leftrightarrow -12\pi < -\pi + 12k\pi \leq 12\pi \Leftrightarrow -11\pi < 12k\pi \leq 13\pi$$

$$\Leftrightarrow -11 < 12k \leq 13 \Leftrightarrow \frac{-11}{12} < k \leq \frac{13}{12} \Leftrightarrow k = 0 \text{ ou } k = 1$$

و:

$$\frac{-\pi}{12} + k\pi \in]-\pi; \pi] \Leftrightarrow x = \frac{-\pi}{12} \text{ ou } x = \frac{-\pi}{12} + \pi = \frac{11\pi}{12}$$

$$S = \left\{ \frac{\pi}{4}; \frac{-3\pi}{4}; \frac{-\pi}{12}; \frac{11\pi}{12} \right\} \text{ بالتالي:}$$

تمرين 4: $A(x) = -2 \cos^2(x) + \sqrt{3} \sin(2x) + 2$

$$A(x) = -2 \cos^2(x) + \sqrt{3} \sin(2x) + 2 = 2(1 - \cos^2(x)) + \sqrt{3} \times 2 \sin x \cos x$$

$$= 2 \sin^2 x + 2\sqrt{3} \sin x \cos x = 4 \sin x \left(\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \right)$$

لدينا:

1

$$A(x) = 4 \sin(x) \left(\frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin(x) \right)$$

$$A(x) = 0 \Leftrightarrow 4 \sin(x) \left(\frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin(x) \right) = 0$$

$$\Leftrightarrow \sin(x) = 0 \text{ ou } \left(\frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin(x) \right) = 0$$

$$\Leftrightarrow x = k\pi / k \in \mathbb{Z} \text{ ou } \cos\left(\frac{\pi}{6}\right) \cos x + \sin\left(\frac{\pi}{6}\right) \sin x = 0$$

$$\Leftrightarrow x = k\pi / k \in \mathbb{Z} \text{ ou } \cos\left(x - \frac{\pi}{6}\right) = 0$$

2

$$\Leftrightarrow x = k\pi / k \in \mathbb{Z} \text{ ou } x - \frac{\pi}{6} = \frac{\pi}{2} + k\pi / k \in \mathbb{Z}$$

$$A(x) = 0 \Leftrightarrow x = k\pi / k \in \mathbb{Z} \text{ ou } x = \frac{2\pi}{3} + k\pi / k \in \mathbb{Z}$$

$$S = \{k\pi / k \in \mathbb{Z}\} \cup \left\{ \frac{2\pi}{3} + k\pi / k \in \mathbb{Z} \right\} \text{ بالتالي:}$$