سلسلة 2	الحساب المثلثي حلول مقترحة	السنة 1 بكالوريا علوم تجريبية

تمرين1 :

$\dfrac{\pi}{4} - \dfrac{\pi}{6} = \dfrac{3\pi}{12} - \dfrac{2\pi}{12} = \dfrac{\pi}{12}$	1

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\tan\left(\frac{\pi}{12}\right) = \frac{\sin\left(\frac{\pi}{12}\right)}{\cos\left(\frac{\pi}{12}\right)} = \frac{\frac{\sqrt{6} - \sqrt{2}}{4}}{\frac{\sqrt{6} + \sqrt{2}}{6}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \frac{\left(\sqrt{6} - \sqrt{2}\right)^2}{6 - 2} = \frac{8 - 2\sqrt{12}}{4} = \frac{8 - 4\sqrt{3}}{4} = 2 - \sqrt{3}$$

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🗨 يمكنك أيضا حساب $\tan\left(\dfrac{\pi}{12}\right)$ باستعمال الخاصية: $\tan(a - b) = \dfrac{\tan a - \tan b}{1 + \tan a \,.\, \tan b}$

تمرين2 :

$$\sqrt{2}\cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right) = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{8}\right)\right)$$ $$\sqrt{2}\cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos\left(\frac{\pi}{8}\right) + \frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{8}\right)\right) = \cos\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{8}\right)$$	لدينا:	1

$\sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2}\cos\left(\frac{\pi}{8}\right)$ منه : $\sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right) = \sqrt{2}\cos\left(\frac{\pi}{4} - \frac{\pi}{8}\right)$	لدينا:	
$\sin\left(\frac{\pi}{8}\right) = \left(\sqrt{2} - 1\right)\cos\left(\frac{\pi}{8}\right)$ منه : $\sin\left(\frac{\pi}{8}\right) = \sqrt{2}\cos\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{8}\right)$	منه :	2
$\boxed{\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1}$ أي : $\dfrac{\sin\left(\frac{\pi}{8}\right)}{\cos\left(\frac{\pi}{8}\right)} = \sqrt{2} - 1$	بالتالي :	

🗨 يمثل التمرين طريقة أخرى لحساب قيمة $\tan\left(\dfrac{\pi}{8}\right)$

تمرين3 :

$\cos a = \dfrac{1}{4}$ ، $\sin b = \dfrac{3}{7}$ ، $a \in \left[0 ; \dfrac{\pi}{2}\right[$ ، $b \in \left]\dfrac{\pi}{2} ; \pi\right]$	تمرين3 :

نعلم أن : $\sin^2 a + \cos^2 a = 1$ منه : $\sin^2 a + \left(\dfrac{1}{4}\right)^2 = 1$ منه : $\sin^2 a = 1 - \dfrac{1}{16} = \dfrac{15}{16}$

وبما أن: $a \in \left[0 ; \dfrac{\pi}{2}\right[$ فإن: $\sin a > 0$ بالتالي: $\sin a = \dfrac{\sqrt{15}}{4}$

نعلم أن : $\sin^2 b + \cos^2 b = 1$ منه : $\left(\dfrac{3}{7}\right)^2 + \cos^2 b = 1$ منه : $\cos^2 b = 1 - \dfrac{9}{49} = \dfrac{40}{49}$

وبما أن: $b \in \left]\dfrac{\pi}{2} ; \pi\right]$ فإن: $\cos b < 0$ بالتالي: $\cos b = \dfrac{-\sqrt{40}}{7}$

$$\cos 2a = 2\cos^2 - 1 = 2 \times \frac{1}{16} - 1 = \frac{1}{8} - 1 = \frac{-7}{8}$$

$$\sin 2a = 2\sin a \cos a = 2 \times \frac{\sqrt{15}}{4} \times \frac{1}{4} = \frac{\sqrt{15}}{8}$$

$$\cos 2b = 2\cos^2 b - 1 = 2 \times \frac{40}{49} - 1 = \frac{80}{49} - 1 = \frac{31}{49}$$

$$\sin 2b = 2\sin b \cos b = 2 \times \frac{3}{7} \times \frac{-\sqrt{40}}{7} = \frac{-6\sqrt{40}}{49}$$

تمرين4 :

$\begin{aligned}\cos(x+y)\cos(x-y) &= (\cos x\cos y - \sin x\sin y)((\cos x\cos y + \sin x\sin y)) \\ &= (\cos x\cos y)^2 - (\sin x\sin y)^2 = \cos^2 x\cos^2 y - \sin^2 x\sin^2 y \\ &= \cos^2 x(1-\sin^2 y) - (1-\cos^2 x)\sin^2 y \\ &= \cos^2 x - \cos^2 x\sin^2 y - \sin^2 y + \cos^2 x\sin^2 y \\ &= \cos^2 x - \sin^2 y\end{aligned}$	1
$\begin{aligned}\sin(x+y)\sin(x-y) &= (\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y) \\ &= (\sin x\cos y)^2 - (\cos x\sin y)^2 \\ &= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y \\ &= (1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y) \\ &= \cos^2 y - \cos^2 x\cos^2 y - \cos^2 x + \cos^2 x\cos^2 y \\ &= \cos^2 y - \cos^2 x\end{aligned}$	2

تمرين5 :

$\sin 3x + \sin 5x = 2\sin\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right)$ $\sin 3x + \sin 5x = 2\sin(4x)\cos(-x)$	$\cos x + \cos 2x = 2\cos\left(\frac{x+2x}{2}\right)\cos\left(\frac{x-2x}{2}\right)$ $\cos x + \cos 2x = 2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{-x}{2}\right)$
$\sin x - \sin\frac{x}{2} = 2\cos\left(\frac{x+\frac{x}{2}}{2}\right)\sin\left(\frac{x-\frac{x}{2}}{2}\right)$ $\sin x - \sin\frac{x}{2} = 2\cos\left(\frac{3x}{4}\right)\sin\left(\frac{x}{4}\right)$	$\cos 3x - \cos 7x = -2\sin\left(\frac{3x+7x}{2}\right)\sin\left(\frac{3x-7x}{2}\right)$ $\cos 3x - \cos 7x = -2\sin(5x)\sin(-2x)$

نذكر بقواعد التعميل :

$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \qquad \cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \qquad \sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$