

حلّول التمرّين

الجذور المربعة

المستوى : الثالثة ثانوي إعدادي

من إعداد الأستاذ : المهدي عيسى

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المملكة المغربية

وزارة التربية الوطنية

والتكوين المهني

الأكاديمية الجهوية للتربية والتكوين

جهة الدار البيضاء الكبرى

تمرين ①

(1) - لنكتب ما يلي بدون جذر مربع :

$$\left(-\sqrt{\sqrt{9}}\right)^2 = \sqrt{9} = 3 \quad ; \quad \sqrt{(-1,5)^2} = 1,5 \quad ; \quad \frac{-\sqrt{225}}{\sqrt{196}} = \frac{-15}{13} \quad ; \quad \sqrt{\left(\frac{-5}{3}\right)^2} = \frac{5}{3} \quad ; \quad \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$\frac{\sqrt{(-18)^2}}{\sqrt{81}} = \frac{18}{9} = 2 \quad ; \quad \sqrt{144} = 12 \quad ; \quad \left(\sqrt{0,001}\right)^2 = 0,001$$

(2) - لنبسط ما يلي :

$$\begin{aligned} B &= \sqrt{100} \times \frac{-2}{3\sqrt{(-4)^2}} + \frac{\sqrt{9}}{\sqrt{16}} \div \sqrt{\left(\frac{3}{4}\right)^2} \\ &= 10 \times \frac{-2}{3 \times 4} + \frac{3}{4} \div \frac{3}{4} \\ &= 10 \times \frac{-2}{12} + \frac{3}{4} \div \frac{3}{4} \\ &= 10 \times \frac{-2}{12} + \frac{3}{4} \times \frac{4}{3} \\ &= 10 \times \frac{-1}{6} + 1 \\ &= 5 \times \frac{-1}{3} + \frac{3}{3} \\ &= \frac{-5}{3} + \frac{3}{3} \\ &= \frac{-2}{3} \end{aligned}$$

$$\begin{aligned} A &= \frac{-5\sqrt{9}}{3} + (\sqrt{-2})^2 - \frac{6}{\sqrt{81}} \\ &= \frac{-5 \times 3}{3} + 2 - \frac{6}{9} \\ &= -5 + 2 - \frac{2}{3} \\ &= -3 - \frac{2}{3} \\ &= \frac{-9}{3} - \frac{2}{3} \\ &= \frac{-11}{3} \end{aligned}$$

$$\begin{aligned} E &= \sqrt{2^2 + 3^2 + 6^2} \\ &= \sqrt{4 + 9 + 36} \\ &= \sqrt{49} \\ &= 7 \end{aligned}$$

$$\begin{aligned} D &= \sqrt{81 - \sqrt{2^2 \times 3^2 + 4^2} - 3} \\ &= 9 - \sqrt{4 \times 9 + 16} - 3 \\ &= 9 - \sqrt{36 + 16} - 3 \\ &= 9 - \sqrt{49} \\ &= 9 - 7 \\ &= 2 \end{aligned}$$

$$\begin{aligned} C &= \sqrt{\frac{\sqrt{81} + \sqrt{(-3)^2}}{\sqrt{9}}} \\ &= \sqrt{\frac{9 + 3}{3}} \\ &= \sqrt{\frac{12}{3}} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

(ج) -- المعادلة $x^2 = -7$ ليس لها حل.

(د) -- المعادلة $x^2 - 3 = -7$ تكافئ على التوالي :

$$x^2 = -7 + 3$$

$$= -4$$

ليس لها حل

(3) -- لنحل المعادلات الآتية :

(أ) -- المعادلة $x^2 = 25$ تكافئ على التوالي :

$$x = -\sqrt{25} \quad \text{أو} \quad x = \sqrt{25}$$

$$x = -5$$

$$x = 5$$

(ب) -- المعادلة $x^2 + 5 = 11$ تكافئ على التوالي :

$$x^2 = 11 - 5$$

$$x^2 = 6$$

$$x = -\sqrt{6} \quad \text{أو} \quad x = \sqrt{6}$$

تصريف ② :

(1) -- لنكتب ما يلي على شكل $a\sqrt{b}$ ، بحيث a و b عدنان حقيقيان موجبان و b أصغر ما يمكن :

$$\frac{1}{2}\sqrt{72} = \frac{1}{2}\sqrt{6^2 \times 2}$$

$$= \frac{1}{2} \times 6\sqrt{2}$$

$$= 3\sqrt{2}$$

$$\sqrt{288} \times \sqrt{75} = \sqrt{12^2 \times 2} \times \sqrt{5^2 \times 3}$$

$$= 12\sqrt{2} \times 5\sqrt{3}$$

$$= 60\sqrt{6}$$

$$\sqrt{\frac{32}{45}} = \sqrt{\frac{4^2 \times 2}{3^2 \times 5}}$$

$$= \sqrt{\left(\frac{4}{3}\right)^2 \times \frac{2}{5}}$$

$$= \frac{4}{3}\sqrt{\frac{2}{5}}$$

$$\sqrt{48} = \sqrt{4^2 \times 3}$$

$$= 4\sqrt{3}$$

$$\sqrt{3^2 \times 5^2 \times 7} = 3 \times 5\sqrt{7}$$

$$= 15\sqrt{7}$$

(2) -- لنحسب ثم نبسط ما يلي :

$$c = -\sqrt{7} + \frac{7 + \sqrt{35}}{\sqrt{7}}$$

$$= \frac{-\sqrt{7}^2}{\sqrt{7}} + \frac{7 + \sqrt{35}}{\sqrt{7}}$$

$$= \frac{-7 + 7 + \sqrt{7 \times 5}}{\sqrt{7}}$$

$$= \frac{\sqrt{7} \times \sqrt{5}}{\sqrt{7}}$$

$$= \sqrt{5}$$

$$b = \sqrt{3} \times \sqrt{27} - \frac{\sqrt{45}}{\sqrt{5}}$$

$$= \sqrt{3} \times \sqrt{3^2 \times 3} - \frac{\sqrt{3^2 \times 5}}{\sqrt{5}}$$

$$= \sqrt{3} \times 3\sqrt{3} - \frac{3\sqrt{5}}{\sqrt{5}}$$

$$= 3\sqrt{3^2} - 3$$

$$= 9 - 3$$

$$= 6$$

$$a = \sqrt{7} \times \sqrt{\frac{25}{7}} + \frac{\sqrt{20}}{\sqrt{5}}$$

$$= \sqrt{7} \times \frac{\sqrt{25}}{\sqrt{7}} + \frac{\sqrt{2^2 \times 5}}{\sqrt{5}}$$

$$= \sqrt{7} \times \frac{5}{\sqrt{7}} + \frac{2\sqrt{5}}{\sqrt{5}}$$

$$= 5 + 2$$

$$= 7$$

(3) -- لنبسط ما يلي :

$$g = 4\sqrt{125} - 2\sqrt{20} - 4\sqrt{5}$$

$$= 4\sqrt{5^2 \times 5} - 2\sqrt{2^2 \times 5} - 4\sqrt{5}$$

$$= 4 \times 5\sqrt{5} - 2 \times 2\sqrt{5} - 4\sqrt{5}$$

$$= (20 - 4 - 4)\sqrt{5}$$

$$= 12\sqrt{5}$$

$$e = \sqrt{99} + 3\sqrt{275} - 2\sqrt{396}$$

$$= \sqrt{3^2 \times 11} + 3\sqrt{5^2 \times 11} - 2\sqrt{6^2 \times 11}$$

$$= 3\sqrt{11} + 3 \times 5\sqrt{11} - 2 \times 6\sqrt{11}$$

$$= (3 + 15 - 12)\sqrt{11}$$

$$= 6\sqrt{11}$$

$$d = 2\sqrt{8} - \sqrt{50} + 3\sqrt{19}$$

$$= 2\sqrt{2^2 \times 2} - \sqrt{5^2 \times 2} + 3\sqrt{19}$$

$$= 2 \times 2\sqrt{2} - 5\sqrt{2} + 3\sqrt{19}$$

$$= 4\sqrt{2} - 5\sqrt{2} + 3\sqrt{19}$$

$$= -\sqrt{2} + 3\sqrt{19}$$

$$\begin{aligned}
 h &= \sqrt{\frac{3}{7}} - 2\sqrt{\frac{12}{28}} + 3\sqrt{\frac{48}{700}} \\
 &= \sqrt{\frac{3}{7}} - 2\sqrt{\frac{2^2 \times 3}{2^2 \times 7}} + 3\sqrt{\frac{4^2 \times 3}{10^2 \times 7}} \\
 &= \sqrt{\frac{3}{7}} - 2\sqrt{\frac{3}{7}} + 3\sqrt{\left(\frac{4}{10}\right)^2 \times \frac{3}{7}} \\
 &= \sqrt{\frac{3}{7}} - 2\sqrt{\frac{3}{7}} + 3 \times \frac{4}{10} \sqrt{\frac{3}{7}} \\
 &= \sqrt{\frac{3}{7}} - 2\sqrt{\frac{3}{7}} + \frac{6}{5} \sqrt{\frac{3}{7}} \\
 &= \left(1 - 2 + \frac{6}{5}\right) \sqrt{\frac{3}{7}} \\
 &= \frac{1}{5} \sqrt{\frac{3}{7}}
 \end{aligned}$$

$$\begin{aligned}
 f &= \sqrt{\frac{80}{49}} - \sqrt{20} + \sqrt{\frac{5}{9}} \\
 &= \frac{\sqrt{4^2 \times 5}}{\sqrt{7^2}} - \sqrt{2^2 \times 5} + \frac{\sqrt{5}}{\sqrt{3^2}} \\
 &= \frac{4\sqrt{5}}{7} - 2\sqrt{5} + \frac{\sqrt{5}}{3} \\
 &= \frac{12\sqrt{5}}{21} - \frac{42\sqrt{5}}{21} + \frac{7\sqrt{5}}{21} \\
 &= \frac{(12 - 42 + 7)\sqrt{5}}{21} \\
 &= \frac{-23\sqrt{5}}{21}
 \end{aligned}$$

∴

تمرين ③

(1) - لنبين أن : $x = y = z$.

لدينا :

$$\begin{aligned}
 z &= \sqrt{3} + \frac{3}{2\sqrt{3}} \\
 &= \sqrt{3} + \frac{3\sqrt{3}}{2\sqrt{3}^2} \\
 &= \sqrt{3} + \frac{3\sqrt{3}}{6} \\
 &= \sqrt{3} + \frac{\sqrt{3}}{2} \\
 &= \frac{3\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \\
 &= \frac{2\sqrt{3} + \sqrt{3}}{2} \\
 &= \frac{3\sqrt{3}}{2}
 \end{aligned}$$

∴

$$\begin{aligned}
 y &= \frac{9}{2\sqrt{3}} \\
 &= \frac{9\sqrt{3}}{2\sqrt{3}^2} \\
 &= \frac{9\sqrt{3}}{6} \\
 &= \frac{3\sqrt{3}}{2}
 \end{aligned}$$

∴

$$\begin{aligned}
 x &= \sqrt{3} + \sqrt{\frac{3}{4}} \\
 &= \sqrt{3} + \frac{\sqrt{3}}{\sqrt{4}} \\
 &= \sqrt{3} + \frac{\sqrt{3}}{2} \\
 &= \frac{2\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \\
 &= \frac{3\sqrt{3}}{2}
 \end{aligned}$$

و بالتالي فإن : $x = y = z$.

(2) - لنزل الجذر المربع من المقام :

$$\begin{aligned}
 \frac{1-\sqrt{3}}{\sqrt{3}+2} &= \frac{(1-\sqrt{3})(\sqrt{3}-2)}{(\sqrt{3}+2)(\sqrt{3}-2)} = \frac{(1-\sqrt{3})(\sqrt{3}-2)}{\sqrt{3}^2 - 2^2} \\
 &= \frac{(1-\sqrt{3})(\sqrt{3}-2)}{3-4} = \frac{(1-\sqrt{3})(\sqrt{3}-2)}{-1}
 \end{aligned}$$

∴

$$\frac{5\sqrt{2}}{3\sqrt{7}} = \frac{5\sqrt{2} \times \sqrt{7}}{3\sqrt{7}^2} = \frac{5\sqrt{14}}{21} \quad \therefore \quad \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}^2} = \frac{2\sqrt{5}}{5}$$

تمرين ④

لنحسب ما يلي :

$$\begin{aligned}
 X &= \frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\
 &= \frac{\sqrt{3}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} - \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} \\
 &= \frac{\sqrt{15}+\sqrt{3}^2}{\sqrt{5}^2-\sqrt{3}^2} - \frac{\sqrt{15}-\sqrt{3}^2}{\sqrt{5}^2-\sqrt{3}^2} \\
 &= \frac{\sqrt{15}+3}{2} - \frac{\sqrt{15}-3}{2} \\
 &= \frac{\sqrt{15}+3-\sqrt{15}+3}{2} \\
 &= \frac{6}{2} = 3
 \end{aligned}$$

$$\begin{aligned}
 Y &= \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}} \\
 &= \frac{1-\sqrt{2}}{1^2-\sqrt{2}^2} + \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} + \frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}^2-\sqrt{4}^2} + \dots + \frac{\sqrt{99}-\sqrt{100}}{\sqrt{99}^2-\sqrt{100}^2} \\
 &= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \frac{\sqrt{3}-\sqrt{4}}{3-4} + \dots + \frac{\sqrt{99}-\sqrt{100}}{99-100} \\
 &= \frac{1-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \frac{\sqrt{3}-\sqrt{4}}{-1} + \dots + \frac{\sqrt{99}-\sqrt{100}}{-1} \\
 &= \frac{1-\sqrt{2}+\sqrt{2}-\sqrt{3}+\sqrt{3}-\sqrt{4}+\sqrt{4}-\dots-\sqrt{99}+\sqrt{99}-\sqrt{100}}{-1} \\
 &= \frac{1-\sqrt{100}}{-1} \\
 &= \frac{1-10}{-1} \\
 &= \frac{-9}{-1} \\
 &= 9
 \end{aligned}$$

تمرين ⑤

(1) - لنثبت أن E عدد صحيح طبيعي :

$$\begin{aligned}
 E &= \frac{5}{\sqrt{3}} + \frac{10}{3+\sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{3}^2} + \frac{10(3-\sqrt{3})}{3^2-\sqrt{3}^2} = \frac{5\sqrt{3}}{3} + \frac{30-10\sqrt{3}}{9-3} = \frac{5\sqrt{3}}{3} + \frac{30-10\sqrt{3}}{6} = \frac{10\sqrt{3}}{6} + \frac{30-10\sqrt{3}}{6} = \frac{10\sqrt{3}+30-10\sqrt{3}}{6} \\
 &= \frac{30}{6} = 5
 \end{aligned}$$

إذن E عدد صحيح طبيعي

(2) - (أ) -- لننشر ما يلي :

$$\begin{aligned}(\sqrt{5}-1)^2 &= \sqrt{5}^2 - 2\sqrt{5} + 1 & (\sqrt{5}+1)^2 &= \sqrt{5}^2 + 2\sqrt{5} + 1 \\ &= 5 - 2\sqrt{5} + 1 & & ; ; & &= 5 + 2\sqrt{5} + 1 \\ &= 6 - 2\sqrt{5} & & & &= 6 + 2\sqrt{5}\end{aligned}$$

(ب) -- لنستنتج تبسيطا لكل من العددين : $\sqrt{6+2\sqrt{5}}$ ثم $\sqrt{6-2\sqrt{5}}$.

لدينا :

$$\sqrt{6-2\sqrt{5}} = \sqrt{(\sqrt{5}-1)^2} = \sqrt{5}-1 \quad \text{و} \quad \sqrt{6+2\sqrt{5}} = \sqrt{(\sqrt{5}+1)^2} = \sqrt{5}+1$$

(ج) - لنحسب العدد : $F = (\sqrt{\sqrt{5}-2} + \sqrt{\sqrt{5}+2})^2$.

لدينا :

$$\begin{aligned}F &= (\sqrt{\sqrt{5}-2} + \sqrt{\sqrt{5}+2})^2 \\ &= \sqrt{(\sqrt{5}-2)^2} + 2 \times \sqrt{\sqrt{5}-2} \times \sqrt{\sqrt{5}+2} + \sqrt{(\sqrt{5}+2)^2} \\ &= (\sqrt{5}-2) + 2\sqrt{(\sqrt{5}-2)(\sqrt{5}+2)} + (\sqrt{5}+2) \\ &= \sqrt{5}-2 + 2\sqrt{\sqrt{5}^2 - 2^2} + \sqrt{5} + 2 \\ &= \sqrt{5}-2 + 2\sqrt{5-4} + \sqrt{5} + 2 \\ &= \sqrt{5}-2 + 2\sqrt{1} + \sqrt{5} + 2 \\ &= \sqrt{5}-2 + 2 + \sqrt{5} + 2 \\ &= 2\sqrt{5} + 2\end{aligned}$$