الجذور المربعة

حلول التمارين

1) $\sqrt{81}=\sqrt{9^2}=9$ ، $\sqrt{144}=\sqrt{12^2}=12$ ، $\sqrt{8^4}=\sqrt{(8^2)^2}=8^2=64$

$\sqrt{2^4\times5^2\times7^6}=\sqrt{(2^2)^2\times5^2\times(7^3)^2}=\sqrt{(2^2\times5\times7^3)^2}=2^2\times5\times7^3$

$\sqrt{\dfrac{49}{36}}=\sqrt{\dfrac{7^2}{6^2}}=\sqrt{\left(\dfrac{7}{6}\right)^2}=\dfrac{7}{6}$

$\sqrt{10^{-6}}=\sqrt{(10^{-3})^2}=10^{-3}$

$\sqrt{0{,}0025}=\sqrt{25\times10^{-4}}=\sqrt{5^2\times(10^{-2})^2}=\sqrt{(5\times10^{-2})^2}=5\times10^{-2}=0{,}05$

$\sqrt{3}\times\sqrt{12}=\sqrt{3\times12}=\sqrt{36}=6$

$\dfrac{\sqrt{24}}{\sqrt{54}}=\sqrt{\dfrac{24}{54}}=\sqrt{\dfrac{6\times4}{6\times9}}=\sqrt{\dfrac{4}{9}}=\sqrt{\left(\dfrac{2}{3}\right)^2}=\dfrac{2}{3}$

2) $(\sqrt{5})^2=5$ ، $(-\sqrt{11})^2=(\sqrt{11})^2=11$ ، $(3\sqrt{5})^3=3^3(\sqrt{5})^3=27(\sqrt{5})^2\sqrt{5}=27\times5\times\sqrt{5}=135\sqrt{5}$

$\left(\dfrac{-\sqrt{7}}{4}\right)^2=\left(\dfrac{\sqrt{7}}{4}\right)^2=\dfrac{7}{16}$

3) $\sqrt{49}=7$ ، $\sqrt{\dfrac{64}{25}}=\dfrac{8}{5}$ ، $\sqrt{2^2\times5^6}=2\times5^3$

4) $\sqrt{\mathbf{a}^2\mathbf{b}^4}=\sqrt{(\mathbf{ab}^2)^2}=\mathbf{ab}^2$

$\sqrt{9\mathbf{a}^4\mathbf{b}^6\mathbf{c}^2}=\sqrt{(3\mathbf{a}^2\mathbf{b}^3\mathbf{c})^2}=3\mathbf{a}^2\mathbf{b}^3\mathbf{c}$

$\sqrt{\dfrac{4\mathbf{a}^6\mathbf{b}^8}{\mathbf{c}^{10}}}=\sqrt{\left(\dfrac{2\mathbf{a}^3\mathbf{b}^4}{\mathbf{c}^5}\right)^2}=\dfrac{2\mathbf{a}^3\mathbf{b}^4}{\mathbf{c}^5}$

5) $\sqrt{63}-\sqrt{112}+\sqrt{700}=\sqrt{7\times9}-\sqrt{7\times16}+\sqrt{7\times100}$

$$= \sqrt{9} \times \sqrt{7} - \sqrt{16} \times \sqrt{7} + \sqrt{100} \times \sqrt{7}$$
$$= 3\sqrt{7} - 4\sqrt{7} + 10\sqrt{7}$$
$$= 9\sqrt{7}$$

$$\sqrt{99} - 10\sqrt{1100} - 6\sqrt{396} = \sqrt{9 \times 11} - 10\sqrt{100 \times 11} - 6\sqrt{36 \times 11}$$
$$= \sqrt{9} \times \sqrt{11} - 10\sqrt{100} \times \sqrt{11} - 6\sqrt{36} \times \sqrt{11}$$
$$= 3\sqrt{11} - 10 \times 10\sqrt{11} - 6 \times 6\sqrt{11}$$
$$= 3\sqrt{11} - 100\sqrt{11} - 36\sqrt{11}$$
$$= -133\sqrt{11}$$

$$\frac{3}{4}\sqrt{48} - 0{,}5\sqrt{108} = \frac{3}{4}\sqrt{16 \times 3} - 0{,}5\sqrt{36 \times 3}$$
$$= \frac{3}{4} \times 4\sqrt{3} - 0{,}5 \times 6\sqrt{3}$$
$$= 3\sqrt{3} - 3\sqrt{3}$$
$$= 0$$

$$\sqrt{\frac{7}{3}} + 4\sqrt{\frac{63}{75}} - 2\sqrt{\frac{28}{27}} = \sqrt{\frac{7}{3}} + 4\sqrt{\frac{9 \times 7}{25 \times 3}} - 2\sqrt{\frac{4 \times 7}{9 \times 3}}$$
$$= \sqrt{\frac{7}{3}} + 4 \times \frac{3}{5} \times \sqrt{\frac{7}{3}} - 2 \times \frac{2}{3} \times \sqrt{\frac{7}{3}}$$
$$= \sqrt{\frac{7}{3}} + \frac{12}{5}\sqrt{\frac{7}{3}} - \frac{4}{3}\sqrt{\frac{7}{3}}$$
$$= \left(1 + \frac{12}{5} - \frac{4}{3}\right) \times \sqrt{\frac{7}{3}}$$
$$= \left(\frac{15 + 36 - 20}{15}\right) \times \sqrt{\frac{7}{3}}$$
$$= \frac{31}{15}\sqrt{\frac{7}{3}}$$

$$\left(3\sqrt{2} - 2\right)^2 = \left(3\sqrt{2}\right)^2 - 2 \times 3\sqrt{2} \times 2 + 2^2$$ **(6**
$$= 9 \times 2 - 12\sqrt{2} + 4$$
$$= 18 - 12\sqrt{2} + 4$$
$$= 22 - 12\sqrt{2}$$

$$\left(\sqrt{15} - \sqrt{30}\right)\left(\sqrt{15} + \sqrt{30}\right) = \left(\sqrt{15}\right)^2 - \left(\sqrt{30}\right)^2$$
$$= 15 - 30$$
$$= -15$$

$$\left(2 + \sqrt{2} + \sqrt{3}\right)\left(2 + \sqrt{2} - \sqrt{3}\right) = \left[\left(2 + \sqrt{2}\right) + \sqrt{3}\right] \times \left[\left(2 + \sqrt{2}\right) - \sqrt{3}\right]$$
$$= \left(2 + \sqrt{2}\right)^2 - \left(\sqrt{3}\right)^2$$

$$= 2^2 + 2\times 2\times\sqrt{2} + \left(\sqrt{2}\right)^2 - 3$$
$$= 4 - 4\sqrt{2} + 2 - 3$$
$$= 3 - 4\sqrt{2}$$

7) $4+2\sqrt{3} = \left(1+\sqrt{3}\right)^2$ ، $9-4\sqrt{5} = \left(2-\sqrt{5}\right)^2$ ، $29-12\sqrt{5} = \left(3-2\sqrt{5}\right)^2$

$\mathbf{a}+4\sqrt{\mathbf{a}}+4 = \left(\sqrt{\mathbf{a}}+2\right)^2$

8) $\sqrt{4+2\sqrt{3}} = \sqrt{\left(1+\sqrt{3}\right)^2} = 1+\sqrt{3}$ ، $\sqrt{9-4\sqrt{5}} = \sqrt{\left(2-\sqrt{5}\right)^2} = \sqrt{\left(\sqrt{5}-2\right)^2} = \sqrt{5}-2$ ، $\sqrt{\mathbf{a}+4\sqrt{\mathbf{a}}+4} = \sqrt{\left(\sqrt{\mathbf{a}}+2\right)^2} = \sqrt{\mathbf{a}}+2$

(لاحظ أن $\sqrt{5}-2 > 0$)

9) $\sqrt{7+2\sqrt{10}} = \sqrt{\left(\sqrt{5}+\sqrt{2}\right)^2} = \sqrt{5}+\sqrt{2}$ ، $\sqrt{21+4\sqrt{5}} = \sqrt{\left(2\sqrt{5}+1\right)^2} = 2\sqrt{5}+1$ ، $\sqrt{11+6\sqrt{2}} = \sqrt{\left(\sqrt{2}+3\right)^2} = \sqrt{2}+3$

10)

$$\frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{\left(\sqrt{5}\right)^2} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

$$\frac{2}{1+\sqrt{5}} = \frac{2\left(\sqrt{5}-1\right)}{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)} = \frac{2\left(\sqrt{5}-1\right)}{\left(\sqrt{5}\right)^2-1^2} = \frac{2\left(\sqrt{5}-1\right)}{5-1} = \frac{2\left(\sqrt{5}-1\right)}{4} = \frac{\sqrt{5}-1}{2}$$

$$\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}+\sqrt{3}\right)}{\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}+\sqrt{3}\right)} = \frac{\left(\sqrt{2}+\sqrt{3}\right)^2}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2} = \frac{\left(\sqrt{2}\right)^2+2\times\sqrt{2}\times\sqrt{3}+\left(\sqrt{3}\right)^2}{2-3} = \frac{2+2\sqrt{6}+3}{-1} = -5-2\sqrt{6}$$

$$\frac{14}{3\sqrt{3}+2\sqrt{5}} = \frac{14\left(3\sqrt{3}-2\sqrt{5}\right)}{\left(3\sqrt{3}+2\sqrt{5}\right)\left(3\sqrt{3}-2\sqrt{5}\right)} = \frac{14\left(3\sqrt{3}-2\sqrt{5}\right)}{(3\sqrt{3})^2-(2\sqrt{5})^2} = \frac{14\left(3\sqrt{3}-2\sqrt{5}\right)}{27-20} = \frac{14\left(3\sqrt{3}-2\sqrt{5}\right)}{7} = 2\left(3\sqrt{3}-2\sqrt{5}\right) = 6\sqrt{3}-4\sqrt{5}$$

$$\frac{2\sqrt{\mathbf{a}}}{1-\sqrt{\mathbf{a}}} = \frac{2\sqrt{\mathbf{a}}\left(1+\sqrt{\mathbf{a}}\right)}{\left(1-\sqrt{\mathbf{a}}\right)\left(1+\sqrt{\mathbf{a}}\right)}$$

$$\frac{1}{\sqrt{\mathbf{a}+\mathbf{b}}-\sqrt{\mathbf{a}}} = \frac{1\left(\sqrt{\mathbf{a}+\mathbf{b}}+\sqrt{\mathbf{a}}\right)}{\left(\sqrt{\mathbf{a}+\mathbf{b}}-\sqrt{\mathbf{a}}\right)\left(\sqrt{\mathbf{a}+\mathbf{b}}+\sqrt{\mathbf{a}}\right)}$$

$$= \frac{2\sqrt{a} + 2\left(\sqrt{a}\right)^2}{1-\left(\sqrt{a}\right)^2}$$
$$= \frac{2\sqrt{a} + 2a}{1-a}$$

$$= \frac{\sqrt{a+b} + \sqrt{a}}{\left(\sqrt{a+b}\right)^2 - \left(\sqrt{a}\right)^2}$$
$$= \frac{\sqrt{a+b} + \sqrt{a}}{a+b-a}$$
$$= \frac{\sqrt{a+b} + \sqrt{a}}{b}$$

(11

$$\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\sqrt{2}-\sqrt{3}+\sqrt{5}} = \frac{\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)\left(\left(\sqrt{2}-\sqrt{3}\right)-\sqrt{5}\right)}{\left(\left(\sqrt{2}-\sqrt{3}\right)+\sqrt{5}\right)\left(\left(\sqrt{2}-\sqrt{3}\right)-\sqrt{5}\right)}$$
$$= \frac{\left(\left(\sqrt{2}-\sqrt{5}\right)+\sqrt{3}\right)\left(\left(\sqrt{2}-\sqrt{5}\right)-\sqrt{3}\right)}{\left(\sqrt{2}-\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$$
$$= \frac{\left(\sqrt{2}-\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}{2+3-2\times\sqrt{2}\sqrt{3}-5}$$
$$= \frac{2+5-2\times\sqrt{2}\times\sqrt{5}-3}{-2\sqrt{6}}$$
$$= \frac{4-2\sqrt{10}}{-2\sqrt{6}}$$
$$= \frac{\left(4-2\sqrt{10}\right)\times\sqrt{6}}{-2\sqrt{6}\times\sqrt{6}}$$
$$= \frac{4\sqrt{6}-2\sqrt{60}}{-2\times 6}$$
$$= \frac{4\sqrt{6}-2\sqrt{60}}{-12}$$
$$= \frac{2\sqrt{60}-4\sqrt{6}}{12}$$
$$= \frac{\sqrt{60}-2\sqrt{6}}{6}$$

$$\frac{1}{\sqrt{5}-\sqrt{3}+2\sqrt{2}} = \frac{1\left(\left(\sqrt{5}-\sqrt{3}\right)-2\sqrt{2}\right)}{\left(\left(\sqrt{5}-\sqrt{3}\right)+2\sqrt{2}\right)\left(\left(\sqrt{5}-\sqrt{3}\right)-2\sqrt{2}\right)}$$
$$= \frac{\sqrt{5}-\sqrt{3}-2\sqrt{2}}{\left(\sqrt{5}-\sqrt{3}\right)^2-\left(2\sqrt{2}\right)^2}$$
$$= \frac{\sqrt{5}-\sqrt{3}-2\sqrt{2}}{\left(\sqrt{5}\right)^2-2\times\sqrt{5}\times\sqrt{3}+\left(\sqrt{3}\right)^2-8}$$
$$= \frac{\sqrt{5}-\sqrt{3}-2\sqrt{2}}{5-2\sqrt{15}+3-8}$$
$$= \frac{\sqrt{5}-\sqrt{3}-2\sqrt{2}}{-2\sqrt{15}}$$

$$= \frac{\left(\sqrt{5}-\sqrt{3}-2\sqrt{2}\right)\times\sqrt{15}}{-2\sqrt{15}\times\sqrt{15}}$$
$$= \frac{\sqrt{5}\times\sqrt{15}-\sqrt{3}\times\sqrt{15}-2\sqrt{2}\times\sqrt{15}}{-2\times 15}$$
$$= \frac{\sqrt{5}\times\sqrt{5}\times\sqrt{3}-\sqrt{3}\times\sqrt{3}\times\sqrt{5}-2\sqrt{30}}{-30}$$
$$= \frac{5\sqrt{3}-3\sqrt{5}-2\sqrt{30}}{-30}$$
$$= \frac{3\sqrt{5}+2\sqrt{30}-5\sqrt{3}}{30}$$

$$\frac{1}{\sqrt{3}+\sqrt{2}-1}+\frac{1}{\sqrt{3}+\sqrt{2}+1} = \frac{\left(\sqrt{3}+\sqrt{2}+1\right)+\left(\sqrt{3}+\sqrt{2}-1\right)}{\left(\sqrt{3}+\sqrt{2}+1\right)\left(\sqrt{3}+\sqrt{2}-1\right)}$$
$$= \frac{2\sqrt{3}+2\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}\right)^2-1^2}$$
$$= \frac{2\sqrt{3}+2\sqrt{2}}{3+2+2\sqrt{6}-1}$$
$$= \frac{2\sqrt{3}+2\sqrt{2}}{2\sqrt{6}+4}$$
$$= \frac{2\left(\sqrt{3}+\sqrt{2}\right)}{2\left(\sqrt{6}+2\right)}$$
$$= \frac{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{6}-2\right)}{\left(\sqrt{6}+2\right)\left(\sqrt{6}-2\right)}$$
$$= \frac{\sqrt{18}-2\sqrt{3}+\sqrt{12}-2\sqrt{2}}{\left(\sqrt{6}\right)^2-2^2}$$
$$= \frac{3\sqrt{2}-2\sqrt{3}+2\sqrt{3}-2\sqrt{2}}{6-4}$$
$$= \frac{\sqrt{2}}{2}$$

12) لمقارنة $\frac{1}{\sqrt{3}-\sqrt{2}}$ و $\frac{3}{\sqrt{3}+\sqrt{2}}$

طريقة 1 : ندرس إشارة فرقهما:

$$\frac{3}{\sqrt{3}+\sqrt{2}}-\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{3\left(\sqrt{3}-\sqrt{2}\right)-\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}$$
$$= \frac{3\sqrt{3}-3\sqrt{2}-\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$
$$= \frac{2\sqrt{3}-4\sqrt{2}}{3-2}$$

$$= \frac{2\left(\sqrt{3}-2\sqrt{2}\right)}{1}$$
$$= 2\left(\sqrt{3}-2\sqrt{2}\right)$$

و لدينا $\left(\sqrt{3}\right)^2 = 3$ و $\left(2\sqrt{2}\right)^2 = 8$

3<8 إذن $2\sqrt{2}$ > $\sqrt{3}$ ومنه $\sqrt{3}-2\sqrt{2}$ <0

و بالتالي $\frac{1}{\sqrt{3}-\sqrt{2}} < \frac{1}{\sqrt{3}+\sqrt{2}}$

طريقة 2 :

نجعل المقام جذريا أولا ثم نقارن

$$\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$$
$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$
$$= \sqrt{3}+\sqrt{2}$$
$$\frac{3}{\sqrt{3}-\sqrt{2}} = \frac{3\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$$
$$= \frac{3\left(\sqrt{3}-\sqrt{2}\right)}{3-2}$$
$$= 3\sqrt{3}-3\sqrt{2}$$

و نقارن $\sqrt{3}+\sqrt{2}$ و $3\sqrt{3}-3\sqrt{2}$ بدراسة إشارة فرقهما مثل الطريقة1 .

13) 1 - لدينا $\left(\sqrt{a}+\sqrt{b}\right)^2 - 4\sqrt{ab} = \left(\sqrt{a}\right)^2 + 2\times\sqrt{a}\times\sqrt{b} + \left(\sqrt{b}\right)^2 - 4\sqrt{ab}$

$$= a + 2\sqrt{ab} + b - 4\sqrt{ab}$$
$$= a - 2\sqrt{ab} + b$$
$$= \left(\sqrt{a}-\sqrt{b}\right)^2$$

و $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$ إذن $\left(\sqrt{a}+\sqrt{b}\right)^2 \geq 4\sqrt{ab}$

2 - $\left(\sqrt{a}+\sqrt{b}\right)^2 - \left(\sqrt{a+b}\right)^2 = a + 2\sqrt{ab} + b - (a+b)$

$$= 2\sqrt{ab}$$

و $2\sqrt{ab} \geq 0$ إذن $\left(\sqrt{a}+\sqrt{b}\right)^2 \geq \left(\sqrt{a+b}\right)^2$

إذن يكون التساوي إذا كان $\sqrt{ab} = 0$ أي $ab = 0$ يعني $a = 0$ أو $b = 0$

14)

لدينا $\left(\sqrt{18+\sqrt{8}}\right)^2 = 18+\sqrt{8}$

$$= 18 + \sqrt{4\times 2}$$
$$= 18 + 2\sqrt{2}$$

$$\left(\sqrt{9+\sqrt{79}} + \sqrt{9-\sqrt{79}}\right)^2 = \left(\sqrt{9+\sqrt{79}}\right)^2 + 2\times\sqrt{9+\sqrt{79}}\times\sqrt{9-\sqrt{79}} + \left(\sqrt{9-\sqrt{79}}\right)^2$$

$$= 9+\sqrt{79}+2\times\sqrt{9^2-\left(\sqrt{79}\right)^2}+9-\sqrt{79}$$
$$=18+2\sqrt{81-79}$$
$$=18+2\sqrt{2}$$

و العددان $\sqrt{18+\sqrt{8}}$ و $\sqrt{9+\sqrt{79}}+\sqrt{9-\sqrt{79}}$ موجبان إذن :

$$\sqrt{18+\sqrt{8}}=\sqrt{9+\sqrt{79}}+\sqrt{9-\sqrt{79}}$$

و بنفس الطريقة

$$\left(\sqrt{6+\sqrt{5}}\right)^2=6+\sqrt{5}$$

$$\sqrt{\frac{6+\sqrt{31}}{2}}+\sqrt{\frac{6-\sqrt{31}}{2}}=\left(\sqrt{\frac{6+\sqrt{31}}{2}}\right)^2+2\times\sqrt{\frac{6+\sqrt{31}}{2}\times\frac{6-\sqrt{31}}{2}}+\left(\sqrt{\frac{6-\sqrt{31}}{2}}\right)^2$$
$$=\frac{6+\sqrt{31}}{2}+2\times\sqrt{\frac{6^2-\left(\sqrt{31}\right)^2}{4}}+\frac{6-\sqrt{31}}{2}$$
$$=\frac{6+\sqrt{31}+2\sqrt{36-31}+6-\sqrt{31}}{2}$$
$$=\frac{12+2\sqrt{5}}{2}$$
$$=6+\sqrt{5}$$

إذن : $\sqrt{6+\sqrt{5}}=\sqrt{\frac{6+\sqrt{31}}{2}}+\sqrt{\frac{6-\sqrt{31}}{2}}$

15)

$$\frac{\mathbf{a}^2+2\mathbf{a}+2\mathbf{a}\sqrt{\mathbf{b}}+\mathbf{b}+2\sqrt{\mathbf{b}}}{\mathbf{a}^2-\mathbf{a}+\mathbf{a}\sqrt{\mathbf{b}}-\sqrt{\mathbf{b}}}=\frac{\left(\mathbf{a}^2+2\mathbf{a}\sqrt{\mathbf{b}}+\mathbf{b}\right)+\left(2\mathbf{a}+2\sqrt{\mathbf{b}}\right)}{\left(\mathbf{a}^2-\mathbf{a}\right)+\left(\mathbf{a}\sqrt{\mathbf{b}}-\sqrt{\mathbf{b}}\right)}$$
$$=\frac{\left(\mathbf{a}+\sqrt{\mathbf{b}}\right)^2+2\left(\mathbf{a}+\sqrt{\mathbf{b}}\right)}{\mathbf{a}\left(\mathbf{a}-1\right)+\sqrt{\mathbf{b}}\left(\mathbf{a}-1\right)}$$
$$=\frac{\left(\mathbf{a}+\sqrt{\mathbf{b}}\right)\left[\left(\mathbf{a}+\sqrt{\mathbf{b}}\right)+2\right]}{\left(\mathbf{a}+\sqrt{\mathbf{b}}\right)\left(\mathbf{a}-1\right)}$$
$$=\frac{\mathbf{a}+\sqrt{\mathbf{b}}+2}{\mathbf{a}-1}$$

16)

$$\sqrt{a}\,\frac{\sqrt{1+\frac{2b}{1+b^2}}+\sqrt{1-\frac{2b}{1+b^2}}}{\sqrt{a+\frac{2ab}{1+b^2}}-\sqrt{a-\frac{2ab}{1+b^2}}}=\sqrt{a}\,\frac{\sqrt{\frac{1+b^2+2b}{1+b^2}}+\sqrt{\frac{1+b^2-2b}{1+b^2}}}{\sqrt{\frac{a+ab^2+2ab}{1+b^2}}-\sqrt{\frac{a+ab^2-2ab}{1+b^2}}}$$
$$=\sqrt{\mathbf{a}}\,\frac{\frac{\sqrt{(1+\mathbf{b})^2}}{\sqrt{1+\mathbf{b}^2}}+\frac{\sqrt{(\mathbf{b}-1)^2}}{\sqrt{1+\mathbf{b}^2}}}{\frac{\sqrt{\mathbf{a}\left(1+\mathbf{b}^2+2\mathbf{b}\right)}}{\sqrt{1+\mathbf{b}^2}}-\frac{\sqrt{\mathbf{a}\left(1+\mathbf{b}^2-2\mathbf{b}\right)}}{\sqrt{1+\mathbf{b}^2}}}$$

$$=\sqrt{a}\,\frac{\dfrac{1+b}{\sqrt{1+b^2}}+\dfrac{b-1}{\sqrt{1+b^2}}}{\dfrac{\sqrt{a}\sqrt{(1+b)^2}}{\sqrt{1+b^2}}-\dfrac{\sqrt{a}\sqrt{(b-1)^2}}{\sqrt{1+b^2}}}$$

$$=\sqrt{a}\,\frac{\dfrac{1+b+b-1}{\sqrt{1+b^2}}}{\dfrac{\sqrt{a}(1+b)-\sqrt{a}(b-1)}{\sqrt{1+b^2}}}$$

$$=\sqrt{a}\,\frac{\dfrac{2b}{\sqrt{1+b^2}}}{\dfrac{\sqrt{a}[(1+b)-(b-1)]}{\sqrt{1+b^2}}}$$

$$=\sqrt{a}\,\frac{\dfrac{2b}{\sqrt{1+b^2}}}{\dfrac{\sqrt{a}(1+b-b+1)}{\sqrt{1+b^2}}}$$

$$=\frac{2b}{\sqrt{1+b^2}}\times\frac{\sqrt{1+b^2}}{2}$$

$$=b$$

17) لحساب A نجعل أولا المقام جذريا في كل حد

$$A=\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\dots\dots+\frac{1}{\sqrt{100}+\sqrt{99}}$$

$$=\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}+\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}+\dots\dots+\frac{\sqrt{100}-\sqrt{99}}{(\sqrt{100}+\sqrt{99})(\sqrt{100}-\sqrt{99})}$$

$$=\frac{\sqrt{2}-1}{2-1}+\frac{\sqrt{3}-\sqrt{2}}{3-2}+\dots\dots+\frac{\sqrt{100}-\sqrt{99}}{100-99}$$

$$=\sqrt{2}-1+\sqrt{3}-\sqrt{2}+\dots\dots+\sqrt{100}-\sqrt{99}$$

$$=-1+\sqrt{100}$$

$$=-1+10=9$$

إذن $A=9$

18) نلاحظ أولا أن AC هي أكبر هذه المسافات
إذن نقارن AB+BC مع AC
لدينا

$$AB+BC=\sqrt{343}+\sqrt{63}$$
$$=\sqrt{49\times7}+\sqrt{9\times7}$$
$$=7\sqrt{7}+3\sqrt{7}$$
$$=10\sqrt{7}$$

و

$$AC=\sqrt{700}=\sqrt{100\times7}=10\sqrt{7}$$

إذن $AB+BC=AC$

و بالتالي النقط A و B و C مستقيمية