الحلول

I

نعلم أن $\sqrt{10^5} = 10^2\sqrt{10}$

لدينا : $3,16 < \sqrt{10} < 3,17$

$$10^2 \times 3,16 < 10^2\sqrt{10} < 10^2 \times 3,17$$

$$316 < \sqrt{10^5} < 317$$

$$316^2 < 10^5 < 317^2$$

إذن الأعداد الصحيحة الطبيعية التي مربعاتها أصغر من 10^5 هي 0 و 1 و 2 و .... و 315 و 316 أي $\boxed{317}$ عددا

II

$$S = \frac{5!}{\sqrt{123}-\sqrt{3}} - \frac{-1}{\sqrt{3!}+\sqrt{7}} + \sqrt{6} - \sqrt{7} - \sqrt{123}$$

$$= \frac{120}{\sqrt{123}-\sqrt{3}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \sqrt{6} - \sqrt{7} - \sqrt{123}$$

$$= \frac{120(\sqrt{123}+\sqrt{3})}{123-3} + \frac{\sqrt{7}-\sqrt{6}}{7-6} + \sqrt{6} - \sqrt{7} - \sqrt{123}$$

$$= \sqrt{123} + \sqrt{3} + \sqrt{7} - \sqrt{6} + \sqrt{6} - \sqrt{7} - \sqrt{123}$$

$$= \boxed{\sqrt{3}}$$

III

$$x < y$$
$$x^2 < y^2$$
$$ax^2 < ay^2$$
$$ax^2 + bxy < ay^2 + bxy$$
$$x(ax+by) < y(ay+bx)$$

(1) $\boxed{\frac{ax+by}{bx+ay} < \frac{y}{x}}$

$$x < y$$
$$x^2 < y^2$$
$$bx^2 < by^2$$
$$bx^2 + axy < by^2 + axy$$
$$x(bx+ay) < y(by+ax)$$

(2) $\boxed{\frac{x}{y} < \frac{ax+by}{bx+ay}}$

من (1) و (2) نستنتج أن : $\boxed{\frac{x}{y} < \frac{ax+by}{bx+ay} < \frac{y}{x}}$

IV

$$P = \left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\cdots\cdots\cdots\left(1-\frac{1}{625}\right)$$

$$= \left(1-\frac{1}{2}\right)\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\cdots\cdots\left(1-\frac{1}{25}\right)\left(1+\frac{1}{25}\right)$$

$$= \left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\cdots\left(1-\frac{1}{25}\right)\right]\left[\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\cdots\left(1+\frac{1}{25}\right)\right]$$

$$= \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \cdots\cdots \times \frac{24}{25}\right)\left(\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \cdots\cdots \times \frac{26}{25}\right)$$

$$= \frac{1}{25} \times \frac{26}{2}$$

$$= \frac{1}{25} \times 13$$

$$= \boxed{\frac{13}{25}}$$

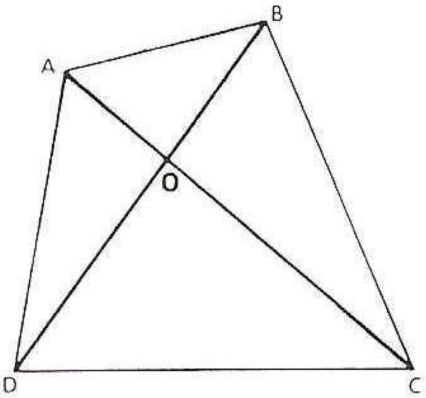
V

لدينا :

$$AB < OA + OB$$
$$BC < OB + OC$$
$$CD < OC + OD$$
$$AD < OA + OD$$

$$p < 2\,(OA + OC) + 2\,(OB + OD)$$
$$p < 2\,AC + 2\,BD$$
$$p < 2\,(AC + BD)$$

$$(1) \qquad \boxed{\frac{1}{2}\,p < AC + BD}$$



في المثلث ABC : $AC < AB + BC$

في المثلث ACD : $AC < AD + CD$

$$2\,AC < AB + BC + AD + CD$$
$$2\,AC < p$$
$$\boxed{AC < \frac{1}{2}\,p}$$

نبرهن بنفس الطريقة أن $\boxed{BD < \frac{1}{2}\,p}$

بما أن $AC < \frac{1}{2}p$ و $BD < \frac{1}{2}p$ فإن : $AC + BD < \frac{1}{2}p + \frac{1}{2}p$

$$(2) \qquad \boxed{AC + BD < p}$$

من (1) و (2) نستنتج أن : $\boxed{\frac{1}{2}\,p < AC + BD < p}$